



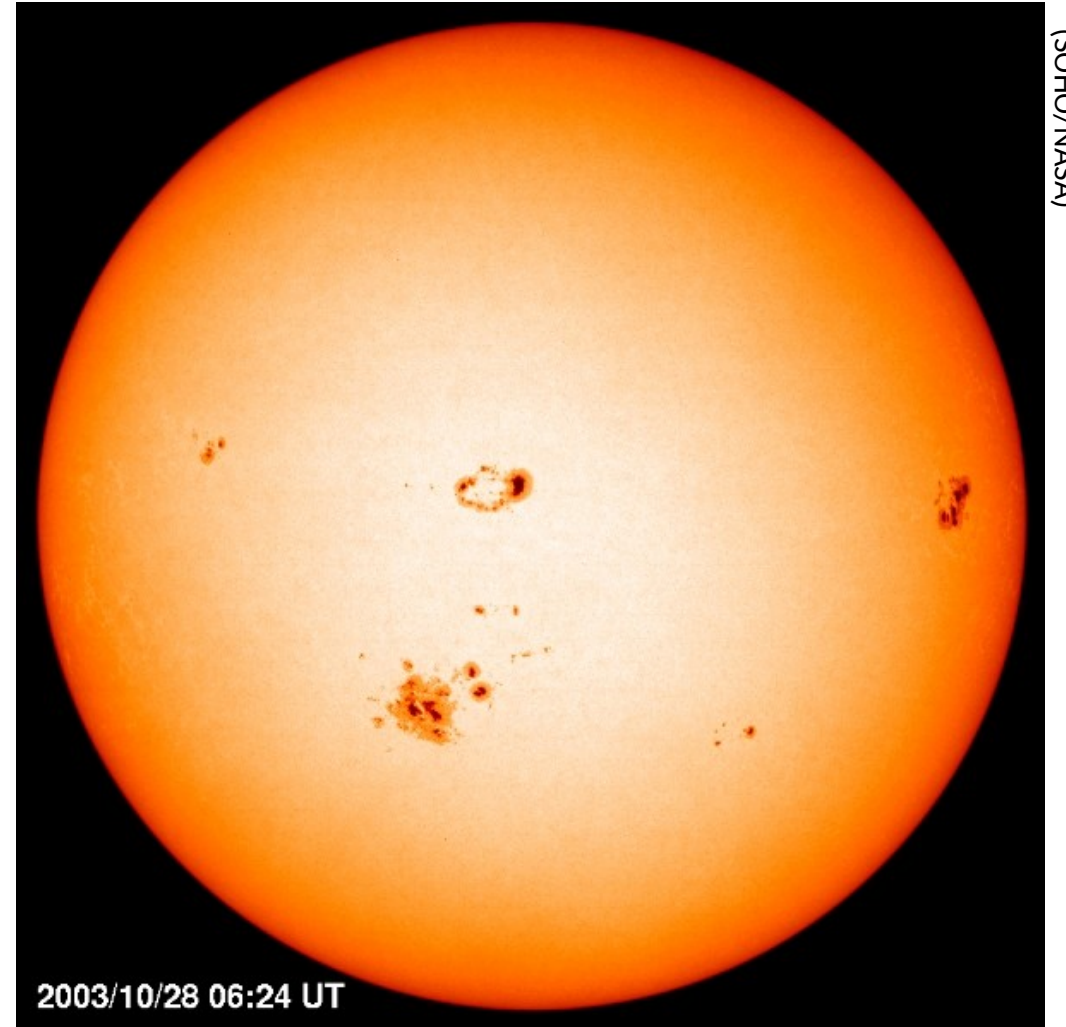
The Sun

Where Are We Going?

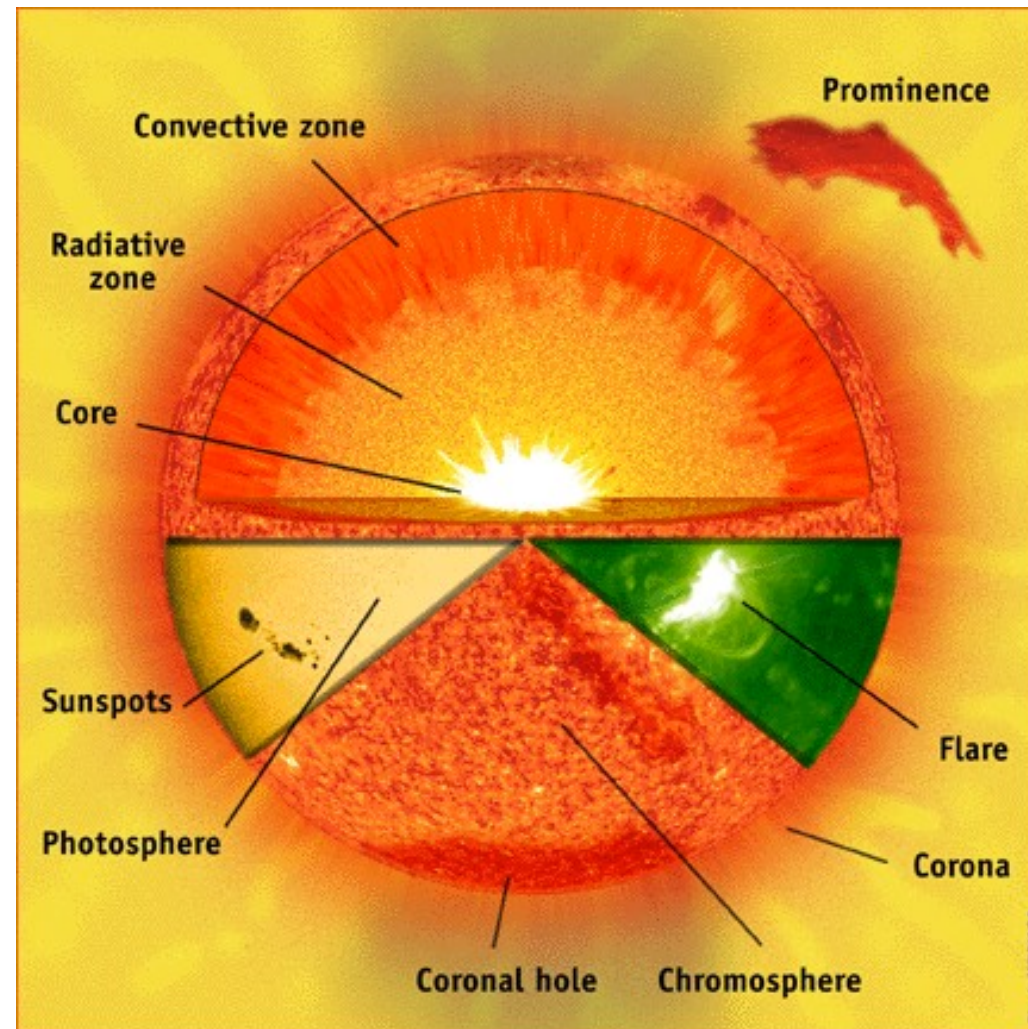
- The Sun (this week)
- WDs (in particular cooling; next week)
- Interacting binaries
- Explosions!
 - Novae and X-ray bursts
 - Thermonuclear supernovae
 - Core-collapse supernovae and neutron stars
 - NS mergers
 - Gamma-ray bursts
- Maybe:
 - Star formation, brown dwarfs
 - Atmospheres
 - Asteroseismology
 - Mass loss

The Sun

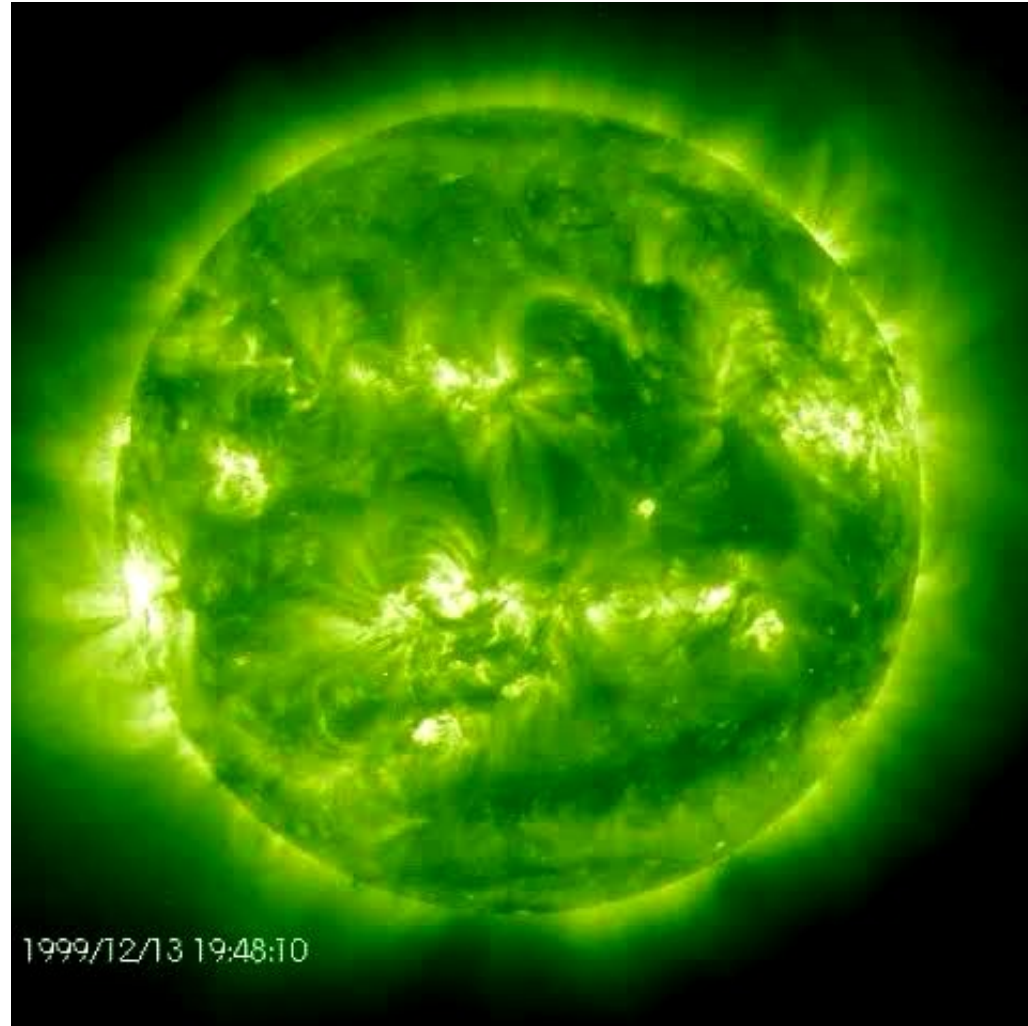
- Notice the limb darkening



- Core: energy produced via thermonuclear reactions
- Radiative and convective layers: transport energy outward
- Photosphere: lower part of solar atmosphere (the part we see)
- Chromosphere: $\sim 10^4$ km thick layer above photosphere—characteristic red color
- Corona: faint tenuous outermost layer of solar atmosphere



Multiwavelength Sun



(Fe XII at 195 angstroms imaged by the EIT instrument on SOHO)

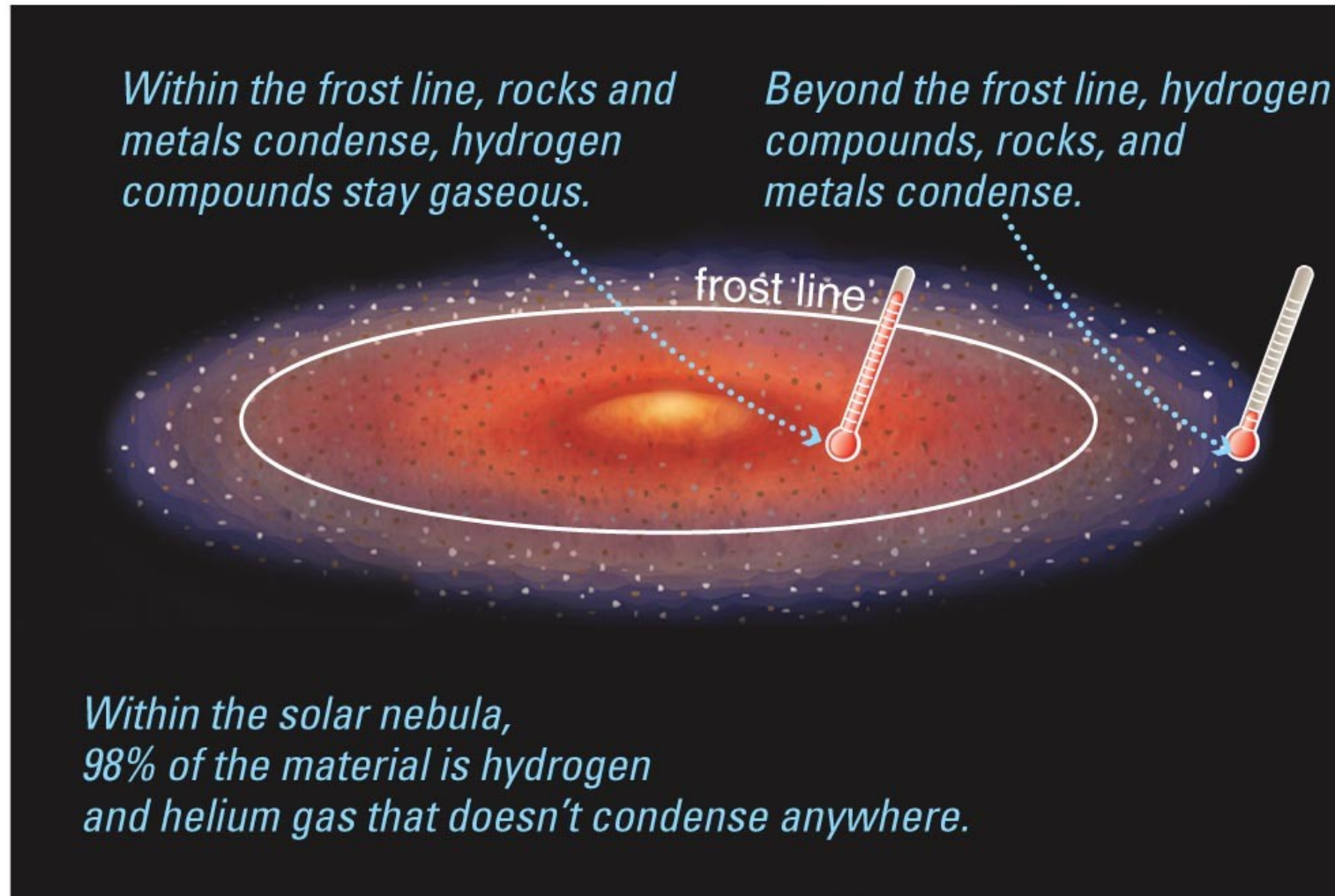
Solar Observations

- We can observe the Sun across the entire electromagnetic spectrum
- We can also get internal information from
 - Neutrinos
 - Helioseismology
- Solar wind measurements tell us about the outer tenuous layers of the Sun.

Basic Properties

- Mass: 1.9891×10^{33} g
 - Measured from gravitational dynamics
- Solar luminosity
 - Varies by $\sim 0.5\%$ —magnetic cycle or surface activity
 - We cannot detect the change in L due to the long timescale evolution of the sun (converting H to He in the core)
- What about abundances and age?
 - Meteorites

Solar System Formation 101



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(from Bennett et al.)

Solar Abundances

- We can get the photosphere abundances by looking at absorption lines in the Sun
- To get the abundances at the time the Sun was formed, we need to use meteorites
 - There are only a handful of meteorites (5 according to Lodders 2003) used for these measurements
 - CI carbonaceous chondrites are used
 - These are the most primitive chondrites
 - This can give us the “Z” that defines solar metallicity
- Note: there is some disagreement in the field about abundances
 - E.g. MESA contains several different scales of solar abundances from different sources

Classification of Meteorites

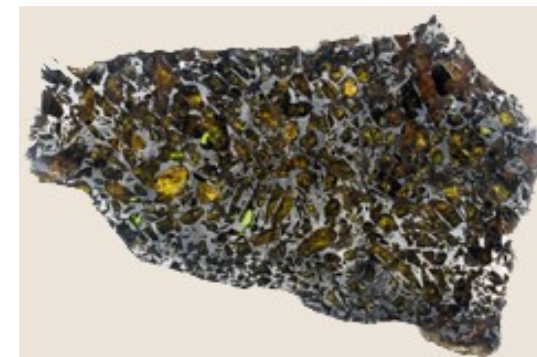
- Iron
 - nearly pure metallic (nickel-iron) of high density (7 g/cm^3)
 - Metals on earth usually oxides
 - They make up only about 5% of observed falls, but are more easily found and recognized than the other types.
- Stony
 - like terrestrial rocks (silicates, carbon compounds)
 - Most common type
 - Usually hard to identify as a meteorite unless the fall was witnessed.
- Stony-iron
 - mixture of iron and stony



Iron meteorite (D. Ball/ASU)



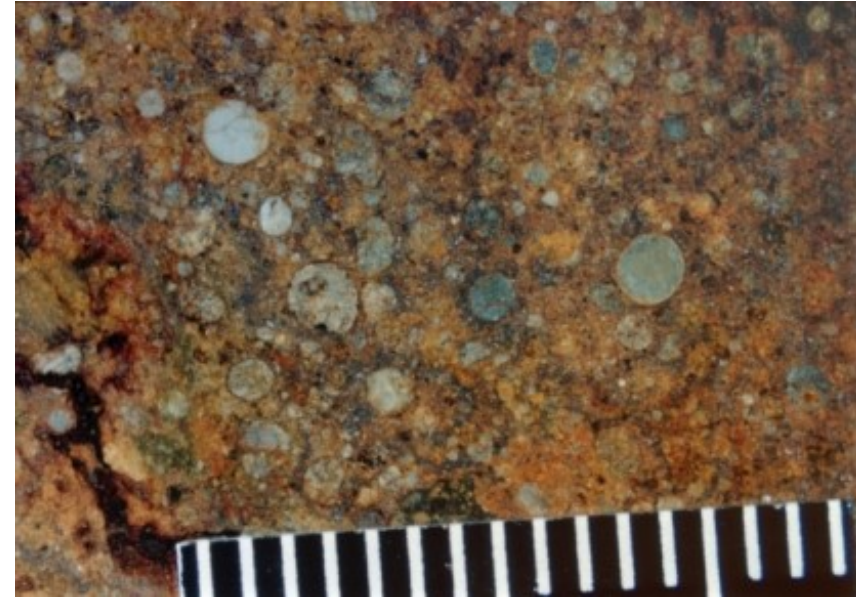
Stony meteorite (D. Ball/ASU)



Stony-iron meteorite D. Ball/ASU

Types of Stony

- Chondrites
 - Most contain chondrules—spherical grains formed from molten material before accretion into parent body
- Achondrites
 - No chondrules
 - Virtually no metals or metal sulphides

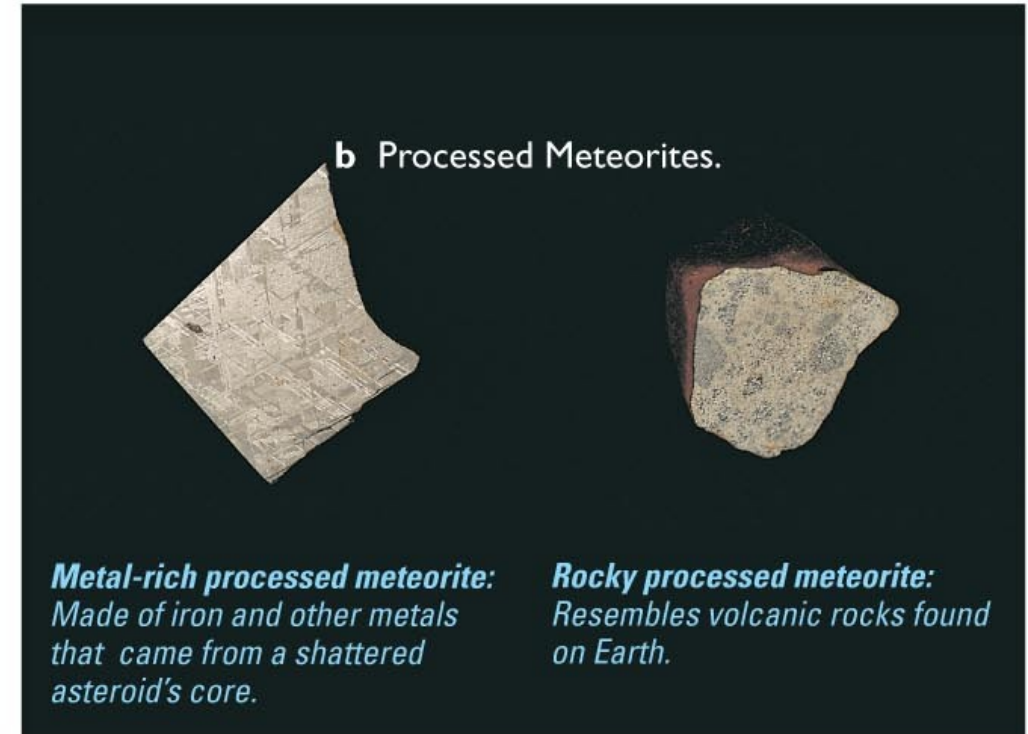
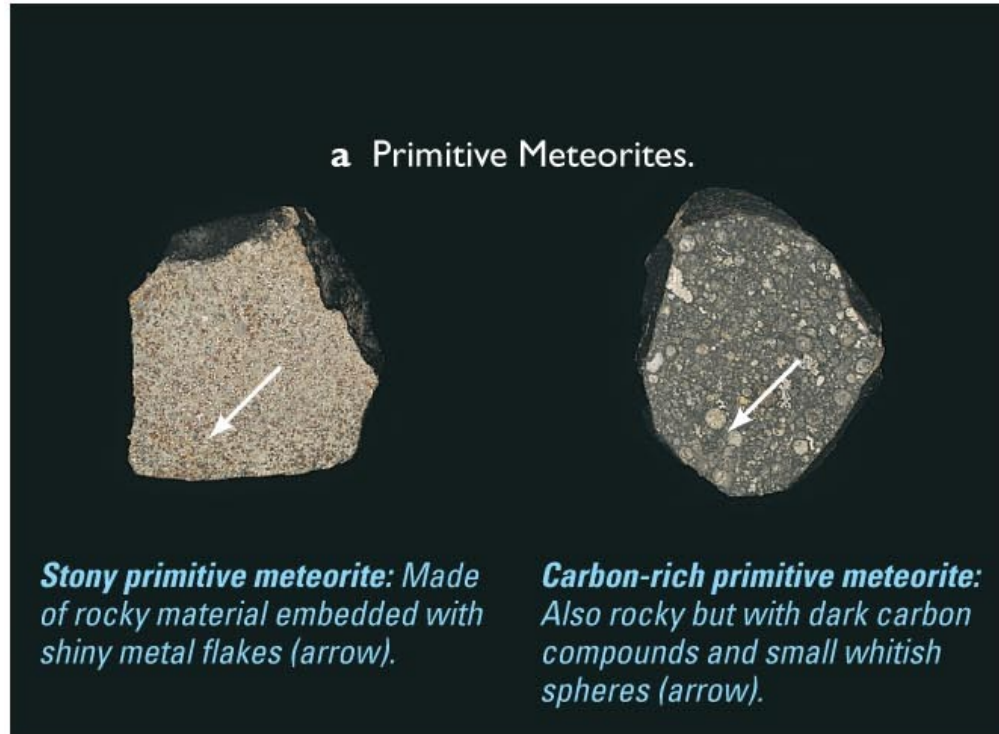


(Wikipedia)

Classification of Meteorites

- Primitive
 - Unchanged since SS formation
 - Only example of primitive rock on Earth
 - Stony: rocky minerals + small fraction of metal flakes
 - Carbon-rich: like stony, with large amounts of carbon compounds
- Processed / Differentiated
 - Significant change since formation (solidified from molten state)
 - Fragment of larger, differentiated object
 - Younger than primitive
 - Metal rich: mostly iron/nickel
 - Rocky: composition resembles terrestrial planet crust/mantle; some with basalts

Classification of Meteorites



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(from Bennett et al.)

Meteoritic Ages

- Age determined by radioactive dating
- Chondrites: 4.3 to 4.5 billion years old
 - Yields age of the solar system
- Some achondrites are 3.2 to 3.7 billion years old
 - Reheating after formation

Heating By Radioactive Decay

- Some meteorites show excess ^{26}Mg in direct proportion to amount of aluminum
- Suggests:



- Originally formed with ^{27}Al (stable) and small amounts of ^{26}Al
 - Half-life: 720,000 years—likely made in nearby supernova
 - Supernova triggered SS formation?
- ^{26}Al would have been important in heating
 - Even small asteroids could melt
 - Allows for idea that large asteroids are parent bodies of meteorites (and not planets)

Solar Abundances

TABLE 2
RECOMMENDED ELEMENTAL ABUNDANCES OF THE PROTO-SUN (SOLAR SYSTEM ABUNDANCES)

Element	$A(\text{El})_0$	$N(\text{El})_0$	Element	$A(\text{El})_0$	$N(\text{El})_0$	Element	$A(\text{El})_0$	$N(\text{El})_0$
H	$\equiv 12$	2.431×10^{10}	Ge	3.70 ± 0.05	120.6	Sm	1.02 ± 0.04	0.2542
He	10.984 ± 0.02	2.343×10^9	As	2.40 ± 0.05	6.089	Sm*	1.02 ± 0.04	0.2554
Li	3.35 ± 0.06	55.47	Se	3.43 ± 0.04	65.79	Eu	0.60 ± 0.04	0.09513
Be	1.48 ± 0.08	0.7374	Br	2.67 ± 0.09	11.32	Gd	1.13 ± 0.02	0.3321
B	2.85 ± 0.04	17.32	Kr	3.36 ± 0.08	55.15	Tb	0.38 ± 0.03	0.05907
C	8.46 ± 0.04	7.079×10^6	Rb	2.43 ± 0.06	6.572	Dy	1.21 ± 0.04	0.3862
N	7.90 ± 0.11	1.950×10^6	Rb*	2.44 ± 0.06	6.694	Ho	0.56 ± 0.02	0.08986
O	8.76 ± 0.05	1.413×10^7	Sr	2.99 ± 0.04	23.64	Er	1.02 ± 0.03	0.2554
F	4.53 ± 0.06	841.1	Sr*	2.99 ± 0.04	23.52	Tm	0.18 ± 0.06	0.03700
Ne	7.95 ± 0.10	2.148×10^6	Y	2.28 ± 0.03	4.608	Yb	1.01 ± 0.03	0.2484
Na	6.37 ± 0.03	5.751×10^4	Zr	2.67 ± 0.03	11.33	Lu	0.16 ± 0.06	0.03572
Mg	7.62 ± 0.02	1.020×10^6	Nb	1.49 ± 0.03	0.7554	Lu*	0.17 ± 0.06	0.03580
Al	6.54 ± 0.02	8.410×10^4	Mo	2.03 ± 0.04	2.601	Hf	0.84 ± 0.04	0.1699
Si	7.61 ± 0.02	$\equiv 1.00 \times 10^6$	Ru	1.89 ± 0.08	1.900	Hf*	0.84 ± 0.04	0.1698
P	5.54 ± 0.04	8373	Rh	1.18 ± 0.03	0.3708	Ta	-0.06 ± 0.03	0.02099
S	7.26 ± 0.04	4.449×10^5	Pd	1.77 ± 0.03	1.435	W	0.72 ± 0.03	0.1277
Cl	5.33 ± 0.06	5237	Ag	1.30 ± 0.06	0.4913	Re	0.33 ± 0.04	0.05254
Ar	6.62 ± 0.08	1.025×10^5	Cd	1.81 ± 0.03	1.584	Re*	0.36 ± 0.04	0.05509
K	5.18 ± 0.05	3692	In	0.87 ± 0.03	0.1810	Os	1.44 ± 0.03	0.6738
K*	5.18 ± 0.05	3697	Sn	2.19 ± 0.04	3.733	Os*	1.44 ± 0.03	0.6713
Ca	6.41 ± 0.03	6.287×10^4	Sb	1.14 ± 0.07	0.3292	Ir	1.42 ± 0.03	0.6448
Sc	3.15 ± 0.04	34.20	Te	2.30 ± 0.04	4.815	Pt	1.75 ± 0.03	1.357
Ti	5.00 ± 0.03	2422	I	1.61 ± 0.12	0.9975	Au	0.91 ± 0.06	0.1955
V	4.07 ± 0.03	288.4	Xe	2.35 ± 0.02	5.391	Hg	1.23 ± 0.18	0.4128
Cr	5.72 ± 0.05	1.286×10^4	Cs	1.18 ± 0.03	0.3671	Tl	0.88 ± 0.04	0.1845
Mn	5.58 ± 0.03	9168	Ba	2.25 ± 0.03	4.351	Pb	2.13 ± 0.04	3.258
Fe	7.54 ± 0.03	8.380×10^5	La	1.25 ± 0.06	0.4405	Pb*	2.12 ± 0.04	3.234
Co	4.98 ± 0.03	2323	Ce	1.68 ± 0.02	1.169	Bi	0.76 ± 0.03	0.1388
Ni	6.29 ± 0.03	4.780×10^4	Pr	0.85 ± 0.03	0.1737	Th	0.16 ± 0.04	0.03512
Cu	4.34 ± 0.06	527.0	Nd	1.54 ± 0.03	0.8355	Th*	0.26 ± 0.04	0.04399
Zn	4.70 ± 0.04	1226	Nd*	1.54 ± 0.03	0.8343	U	-0.42 ± 0.04	9.306×10^{-3}
Ga	3.17 ± 0.06	35.97				U*	$+0.01 \pm 0.04$	24.631×10^{-3}

NOTE.—Values for elements marked with an asterisk are abundances 4.55×10^9 yr ago. Mass fractions for proto-Sun: $X_0 = 0.7110$, $Y_0 = 0.2741$, $Z_0 = 0.0149$, and $X_0/Z_0 = 0.0210$. The astronomical log scale and the cosmochemical abundance scale by number are coupled by $A(\text{El})_0 = \log[N(\text{El})] + 1.614$.

(Lodders 2003)

Solar Abundances

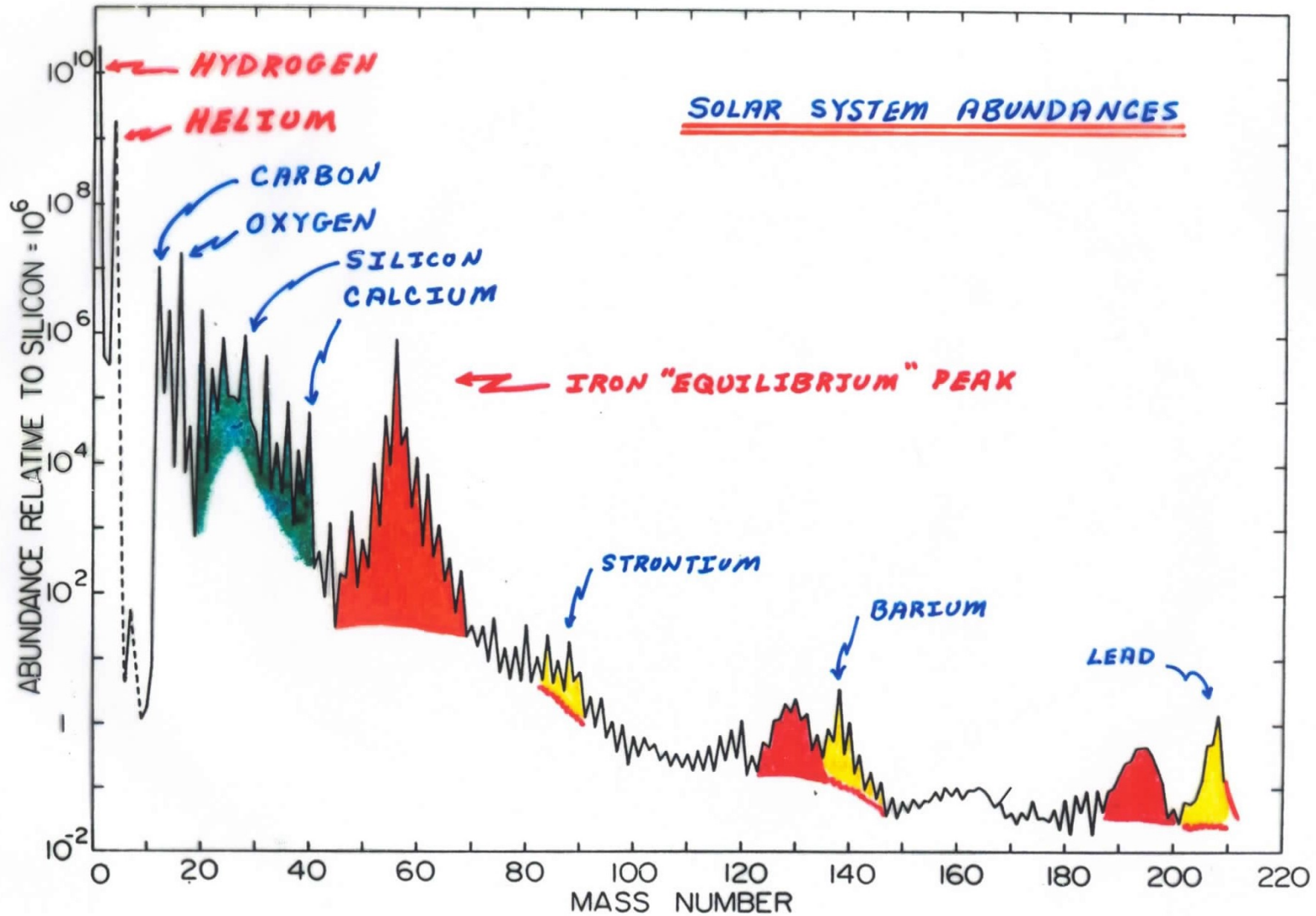
- Note the funny scale:

$$A(X) = \log_{10} \left(\frac{n(X)}{n(H)} \right) + 12$$

- Normalized such that $n(H) = 10^{12}$

- After H, He, we see that O and C are the next most abundant

Solar Abundances



(Jim Truran)

Solar Abundances

- Constrain Y from evolutionary sequences and comparing to the present-day luminosity and radius
 - Heavily dependent on the input microphysics
 - Over time, settling of He from the surface occurs \rightarrow photosphere value not representative of the interior
- Accepted values (Bahcall et al. 2001):
 - $X = 0.7169$, $Y = 0.2656$, $Z = 0.01757$ (no diffusion)
 - $X = 0.7077$, $Y = 0.2735$, $Z = 0.0188$ (with diffusion)
- Result: we get the luminosity, mass, radius, effective T , initial composition, and age reliably

ZAMS Sun

- Start with the mass and composition and evolve to the present day
 - Assumes negligible mass loss, ignores rotation and magnetic fields
 - Asteroseismology measurements: mass loss on the red giant branch is very small ($\sim 5\%$) for low mass stars
 - Free parameters:
 - He abundance (metals/H fixed by meteoritic studies)
 - Convection parameters (in particular the mixing length)
 - Ex: models presented in your text (ignoring gravitational settling of He) need
 - $Y = 0.288$
 - $\alpha = 1.2$
 - Open question: Should the same mixing length be used for all time and all locations in the star

ZAMS Sun

- Solar ZAMS structure

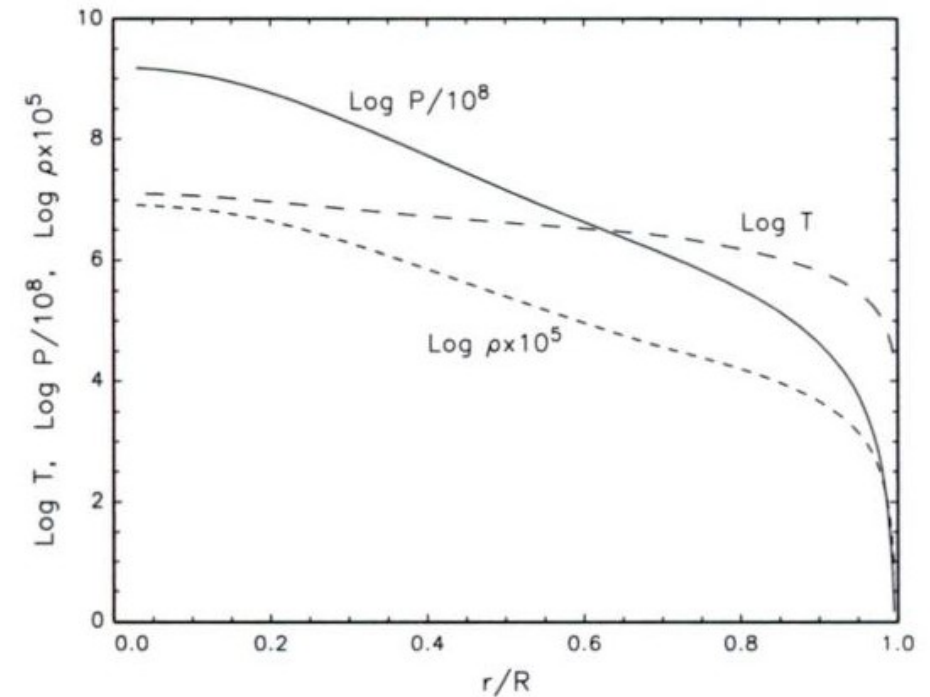


Fig. 9.1. Shown are the runs of pressure, temperature, and density for a model of the zero-age sun. Note that the pressure has been multiplied by 10^{-8} and the density by 10^5 . The abscissa is the relative radius r/\mathcal{R} where $\mathcal{R} = 0.886 \mathcal{R}_{\odot}$.

(HKT)

ZAMS Sun

- L and M as a function of r
 - ZAMS sun has $R = 0.886 R_{\odot}$, $L = 0.725 L_{\odot}$
 - Note: present-day Sun has luminosity and mass peaked more toward the center—this is due to evolution (and contraction of the core)

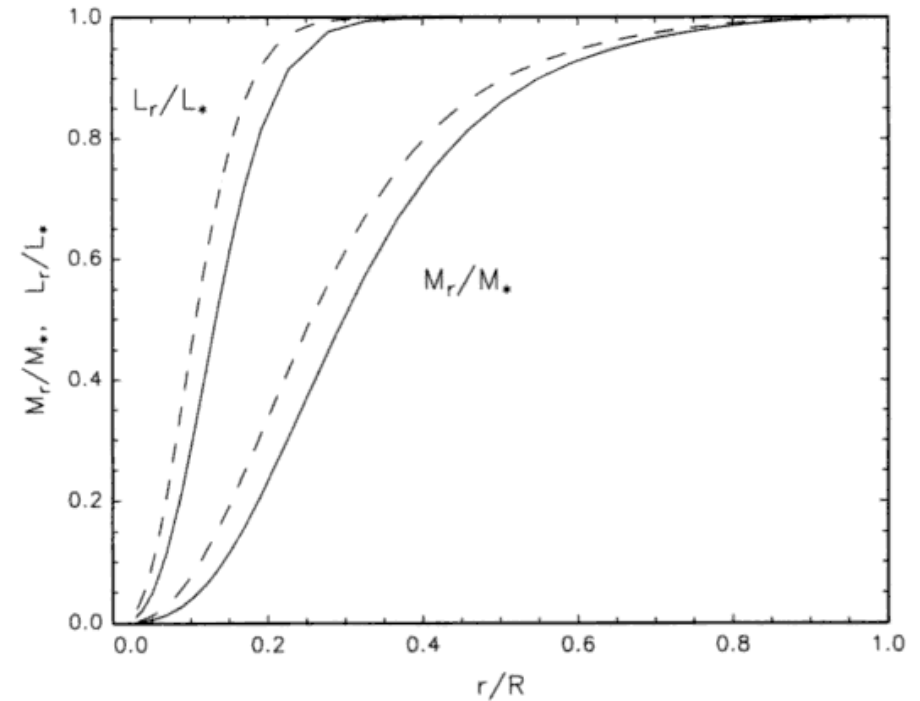


Fig. 9.2. Relative luminosity, $\mathcal{L}_r/\mathcal{L}_*$, and mass, $\mathcal{M}_r/\mathcal{M}_*$, are plotted versus relative radius for the ZAMS (solid lines) and present-day sun (dashed lines). Total radius and mass for the ZAMS model are $\mathcal{R} = 0.886 \mathcal{R}_{\odot}$ and $\mathcal{L} = 0.725 \mathcal{L}_{\odot}$.

(HKT)

Evolution off ZAMS

- Recall the dimensional analysis (HW 2)

- Ideal gas:

$$P \sim \frac{\rho T}{\mu}$$

- HSE:

$$P \sim \frac{GM^2}{R^4}$$

- Radiation transport

$$L \sim \frac{T^4 R^4}{\kappa M}$$

- All together (w/ Kramer's opacity):

$$L \sim \frac{M^{5.5} \mu^{7.5}}{Z(1+X)} R^{-0.5}$$

- Two assumptions:

- Radiation pressure is negligible in the Sun
- Radius don't not change much as the Sun evolves

- Luminosity evolves via:

$$\frac{L(t)}{L(0)} = \left[\frac{\mu(t)}{\mu(0)} \right]^{7.5}$$

Solar Thermodynamics

- For solar mix, the conditions in the center of the Sun are:

```
temp = 1.5000E+07  den = 1.5000E+02  abar = 1.2715E+00  zbar = 1.0935E+00

helm:  total      ion      e- & e+    radiation  coulomb
pres = 3.1592E+17  1.4713E+17  1.6866E+17  1.2767E+14 -1.0085E+15
ener = 3.1659E+15  1.4713E+15  1.6921E+15  2.5535E+12 -2.0171E+13
entr = 1.2412E+09  9.6797E+08  2.7300E+08  2.2698E+05 -4.2249E+05

dp/dd= 2.1568E+15  dp/dt= 2.0309E+10  dp/da= -2.5444E+17  dp/dz= 1.6131E+17
de/dd= 5.0121E+11  de/dt= 2.0412E+08  de/da= -2.5490E+15  de/dz= 1.6185E+15
ds/dd= -9.0264E+05  ds/dt= 1.3608E+01  ds/da= -7.9253E+08  ds/dz= 1.8585E+08

dpepd= 1.1760E+15  dpept= 1.0467E+10  dpepa= -1.3873E+17  dpepz= 1.6131E+17
deepd= 5.1824E+11  deepst= 1.0536E+08  deepa= -1.3919E+15  deepz= 1.6185E+15
dsepd= -4.6519E+05  dsept= 7.0238E+00  dsepa= -1.5983E+08  dsepz= 1.8585E+08

maxw1= 2.2204E-16  maxw2= 2.2204E-16  maxw3= 2.2204E-16
xne = 7.7686E+25  xnp = 0.0000E+00  xnem = 7.7686E+25  xni = 7.1042E+25
dxned= 5.1790E+23  dxnet= -8.0400E+03  dxnea= -6.1096E+25  dxnez= 7.1042E+25
eta = -1.1921E+00  etap = 0.0000E+00
detad= 7.3121E-03  detat= -1.0990E-07  detaa= -8.6259E-01  detaz= 1.0030E+00
cp = 3.3162E+08  cv = 2.0412E+08  plasg= 8.8934E-02
gam1 = 1.6637E+00  gam2 = 1.6630E+00  gam3 = 1.6633E+00  csond= 5.9194E+07
```

$\eta > 1$ for degeneracy (this is electron chemical potential / kT)

Using Timmes Helmeos

Side Note: EOS

- Popular astrophysical EOS: helmeos http://cococubed.asu.edu/code_pages/eos.shtml

- Uses a table of electron/positron contributions

- Composition factors out of the table, since

$$n_e = \frac{Z}{A} \frac{\rho}{m_u}$$

- Tabulates helmholtz free energy, F , in terms of T , ρ

$$F = e - Ts, \quad dF = -s dT + \frac{P}{\rho^2} d\rho$$

- All thermodynamic quantities and derivatives can be found by differentiating F

$$P = \rho^2 \left. \frac{\partial F}{\partial \rho} \right|_T \quad s = - \left. \frac{\partial F}{\partial T} \right|_\rho \quad e = F + Ts$$

- Interpolant (2-d) itself differentiated to guarantee consistency

- Radiation and ions computed analytically

Evolution off the ZAMS

- Mean molecular weight (complete ionization):

$$\mu = \left(\sum_k \frac{X_k}{A_k} (Z_k + 1) \right)^{-1} \sim \frac{4}{3 + 5X(t)}$$

- How does X change with time?

- Number of reactions / s:

$$N_{\text{react}} = \frac{L}{E_{\text{react}}} \quad E_{\text{react}} = 0.03m_p c^2 - \text{neutrinos}$$

- Mass of H destroyed / s:

$$\frac{\Delta m_H}{\Delta t} = -4m_p N_{\text{react}} = -L \frac{4m_p}{E} = -\frac{L}{Q} \quad (Q = 6 \times 10^{18} \text{ erg /g})$$

Evolution off the ZAMS

- Define an average H mass fraction in the Sun:

$$X \equiv \frac{m_H}{M_\odot}$$

- Then:

$$\frac{dX}{dt} = -\frac{L_\odot}{QM_\odot}$$

- We want to use this to find the evolution of L :

$$\frac{d\mu}{dt} = -\frac{5}{4} [\mu(t)]^2 \frac{dX}{dt} = \frac{5}{4} [\mu(t)]^2 \frac{L}{QM}$$

$$\frac{dL}{dt} = \frac{15}{2} L(0) \left[\frac{\mu(t)}{\mu(0)} \right]^{15/2-1} \frac{1}{\mu(0)} \frac{d\mu}{dt}$$

Evolution off the ZAMS

- Expressing this only in terms of $L(t)$, we find:

$$\frac{dL}{dt} = \frac{75}{8} \frac{\mu(0)}{QM} \frac{L(t)^{1+17/15}}{L(0)^{17/15-1}}$$

- This has the solution:

$$L(t) = L(0) \left[1 - \frac{85}{8} \frac{\mu(0)L(0)}{MQ} t \right]^{-15/17}$$

- Putting in solar #s, we find

$$\frac{L(t)}{L_{\odot}} \approx \frac{L(0)}{L_{\odot}} \left[1 - 0.3 \frac{L(0)}{L_{\odot}} \frac{t}{t_{\odot}} \right]^{-15/17}$$

- Constant evaluated by matching to present-day luminosity:

$$L(0) \sim 0.79L_{\odot}$$

Evolution off the ZAMS

- When the Sun formed, its luminosity was only 80% of the present-day luminosity
- Recall that we estimated the temperature of the earth based on the radiation it receives:
- Life formed > 3 billion years ago, and likely required liquid water, so there was likely a different atmosphere in place then.

$$P_{\text{abs}} = \epsilon \pi R_{\oplus}^2 \frac{L_{\odot}}{4\pi d^2} = P_{\text{emitted}} = 4\pi R_{\oplus}^2 \sigma T^4$$

$$T = \left[\epsilon \frac{L_{\odot}}{16\pi\sigma d^2} \right]^{1/4}$$

Present Day Sun

- Recall that the main sequence is a band
- From ZAMS until we leave the main sequence is $\sim 10^{10}$ yr for the Sun
 - Inner core has changed a lot since ZAMS
 - Recall:
 - As H becomes He, pressure support is removed
 - Sun's core contracts, gravitational PE released
 - Increased T \rightarrow increased fusion rate
 - Luminosity gradient steepens
- Current central conditions:
 - Density: 146 g cm^{-3}
 - Temperature: $1.53 \times 10^7 \text{ K}$

Present Day Sun

- Mass fractions are strongly radially dependent
- CNO is now $\sim 7\%$ of the energy generation

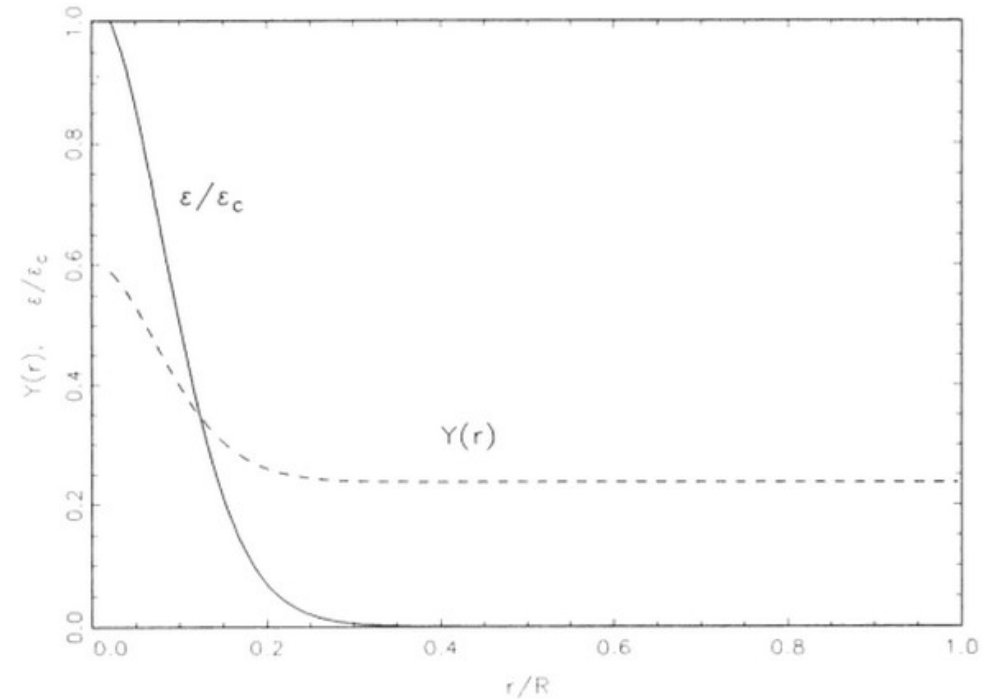


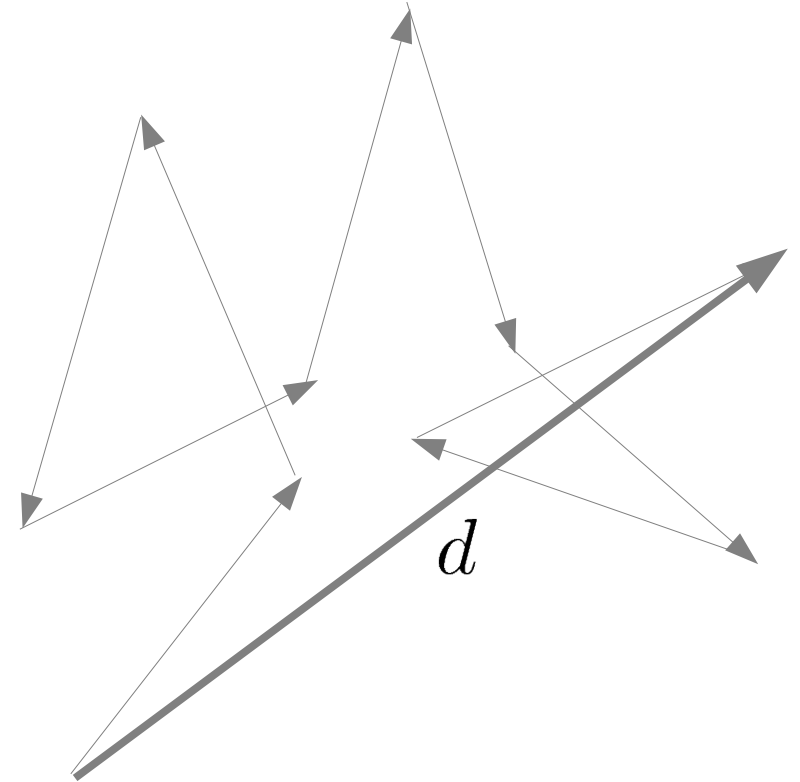
FIGURE 8.4. Because of hydrogen-burning in the radiative solar core, the mass fraction, Y , of helium in the present-day sun has increased while the mixture becomes less hydrogen-rich. Also shown is the energy generation rate $\epsilon(r)$ as compared to its central value ϵ_c .

(HKT)

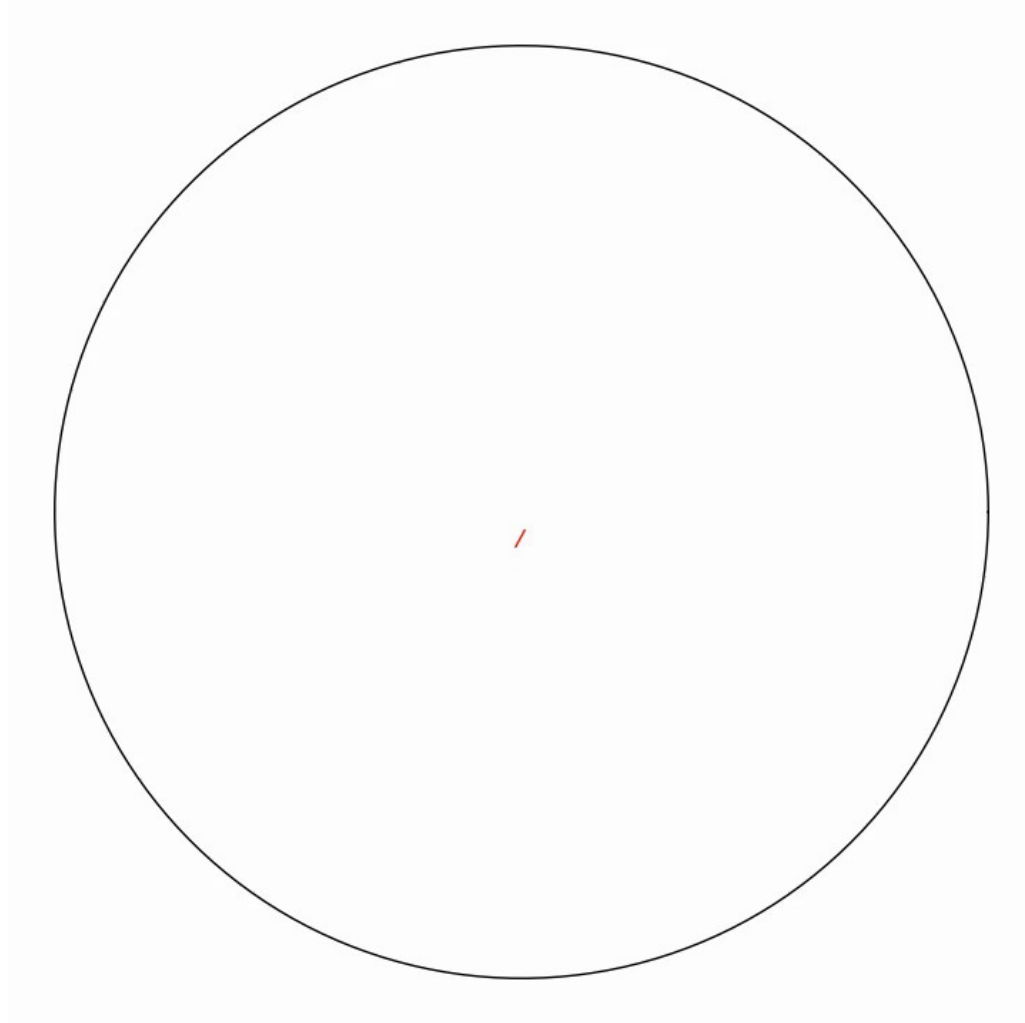
Aside: Random Walk

(Shu Ch. 5 and Carroll and Ostlie Ch 9)

- What's the timescale for photon diffusion to carry energy from the core?
 - Photons moving outward from center of the Sun are continually absorbed and re-emitted.
 - Photon path to the surface of the Sun is a series of short jumps
- Jump length is the mean free path
- This is a random walk process



Aside: Random Walk



Aside: Random Walk

- Net displacement is just the sum of the individual hops:

$$\vec{d} = \vec{l}_1 + \vec{l}_2 + \vec{l}_3 + \dots + \vec{l}_N$$

- Distance traveled is vector length:

$$\begin{aligned} \vec{d} \cdot \vec{d} &= \vec{l}_1 \cdot \vec{l}_1 + \vec{l}_1 \cdot \vec{l}_2 + \dots + \vec{l}_1 \cdot \vec{l}_N \\ &\quad + \vec{l}_2 \cdot \vec{l}_1 + \vec{l}_2 \cdot \vec{l}_2 + \dots + \vec{l}_2 \cdot \vec{l}_N \\ &\quad + \dots + \\ &\quad + \vec{l}_N \cdot \vec{l}_1 + \vec{l}_N \cdot \vec{l}_2 + \dots + \vec{l}_N \cdot \vec{l}_N \\ &= \sum_{i=1}^N \sum_{j=1}^N \vec{l}_i \cdot \vec{l}_j \end{aligned}$$

- Sum consists of N dot products of a segment with itself and lots of cross-terms

$$d^2 = Nl^2 + l^2 \sum_{i=1}^N \sum_{j=1, j \neq i}^N \cos \theta_{ij}$$

- Angles are randomly distributed.
 - Large number of hops, sum of the cosine terms $\rightarrow 0$:

$$d = l\sqrt{N}$$

- Photon leaving the Sun: each step's length is the mean free path, L, total # of steps is

$$N = \frac{R_{\odot}^2}{L^2}$$

Aside: Random Walk

- Average mean free path in the Sun is ~ 0.5 cm, so # of steps is

$$N = \frac{(7 \times 10^{10} \text{ cm})^2}{(0.5 \text{ cm})^2} = 2 \times 10^{22}$$

- Time for a single step is

$$t_l = \frac{L}{c} = \frac{0.5 \text{ cm}}{3 \times 10^{10} \text{ cm s}^{-1}} = 1.7 \times 10^{-11} \text{ s}$$

- Amount of time for a photon to random walk out of the Sun:

$$\begin{aligned} t &= t_l N = 2 \times 10^{22} \cdot 1.7 \times 10^{-11} \text{ s} \\ &= 3.4 \times 10^{11} \text{ s} = 10\,000 \text{ yr} \end{aligned}$$

- True value is a bit higher because the mean free path is smaller in the center of the Sun.

- For comparison, the light-travel time across a solar radius is

$$t = R_{\odot}/c = 2.3 \text{ s}$$

Chromosphere

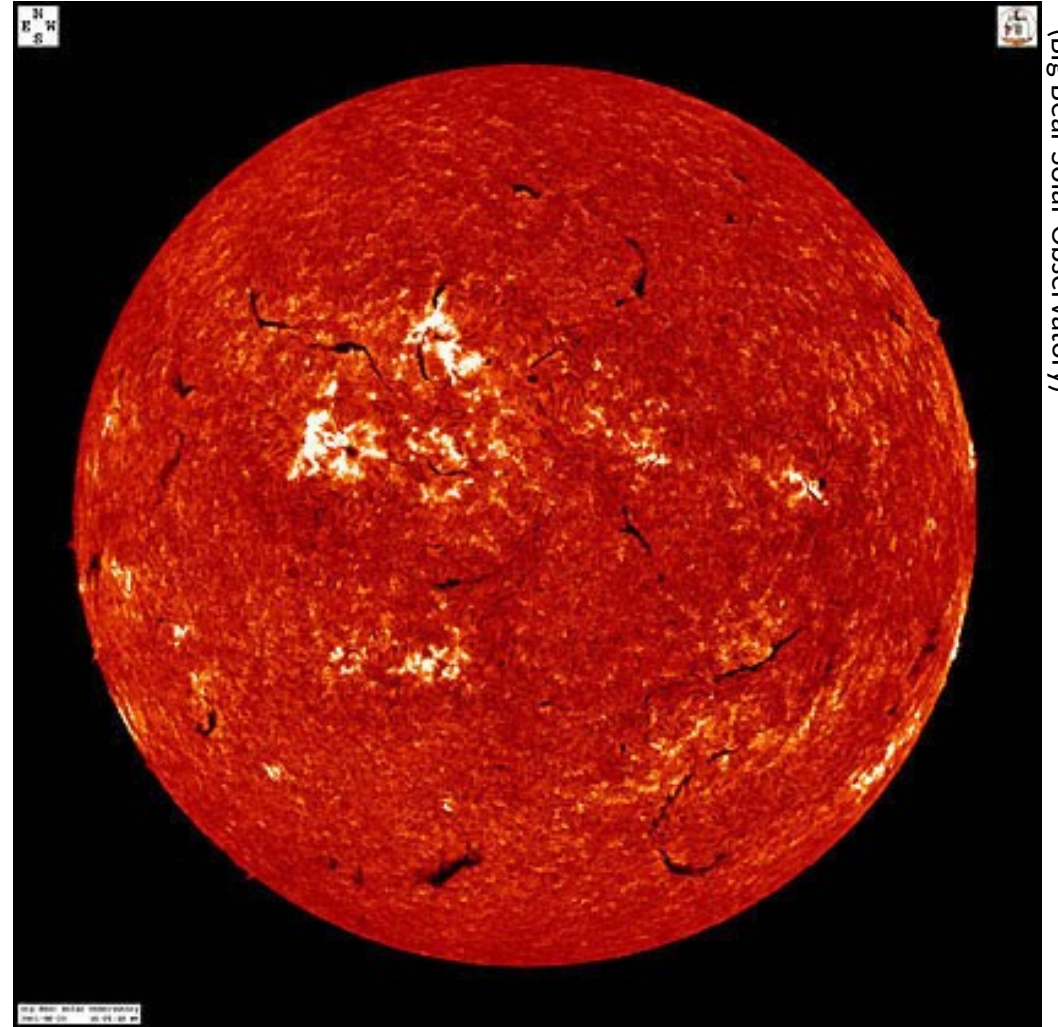
- Most of the light we see comes from the photosphere, it accounts for only 0.1% of the radius of the Sun.
- Moving out from the photosphere: chromosphere.
 - Density is just 1/10000th of photosphere
 - Usually we do not see the chromosphere, since it is optically thin
 - Just before and after an eclipse, we can see it.
- If we take a spectra during the eclipse, we see very strong H α emission (not absorption)
 - Indicating a temperature of 15000 K
- Interestingly, as we move outward from the photosphere, the temperature begins to increase!

Chromosphere

- Looking in $H\alpha$, we see large scale features (supergranules)



(Wikipedia)



(Big Bear Solar Observatory)

Corona

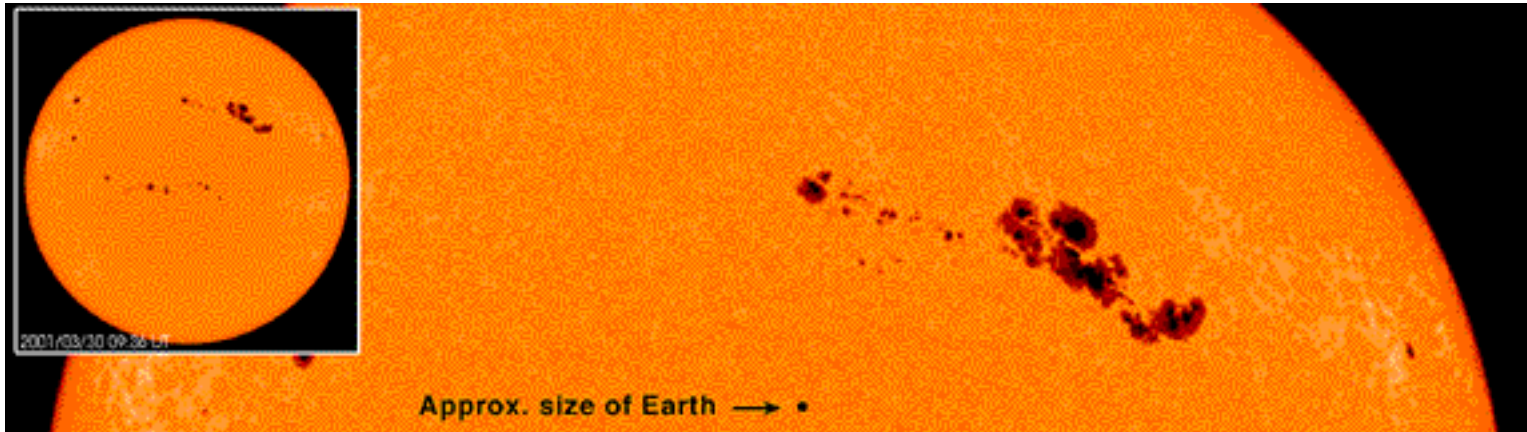
- Above the chromosphere: corona
 - Hotter, more tenuous
 - Not in LTE
 - Very small optical depth
 - Only seen during eclipses
- $T \sim 2$ million K
- Source of Sun's radio emission and X-ray (photosphere's blackbody radiation is negligible at these wavelengths)



(Wikipedia)

Sunspots

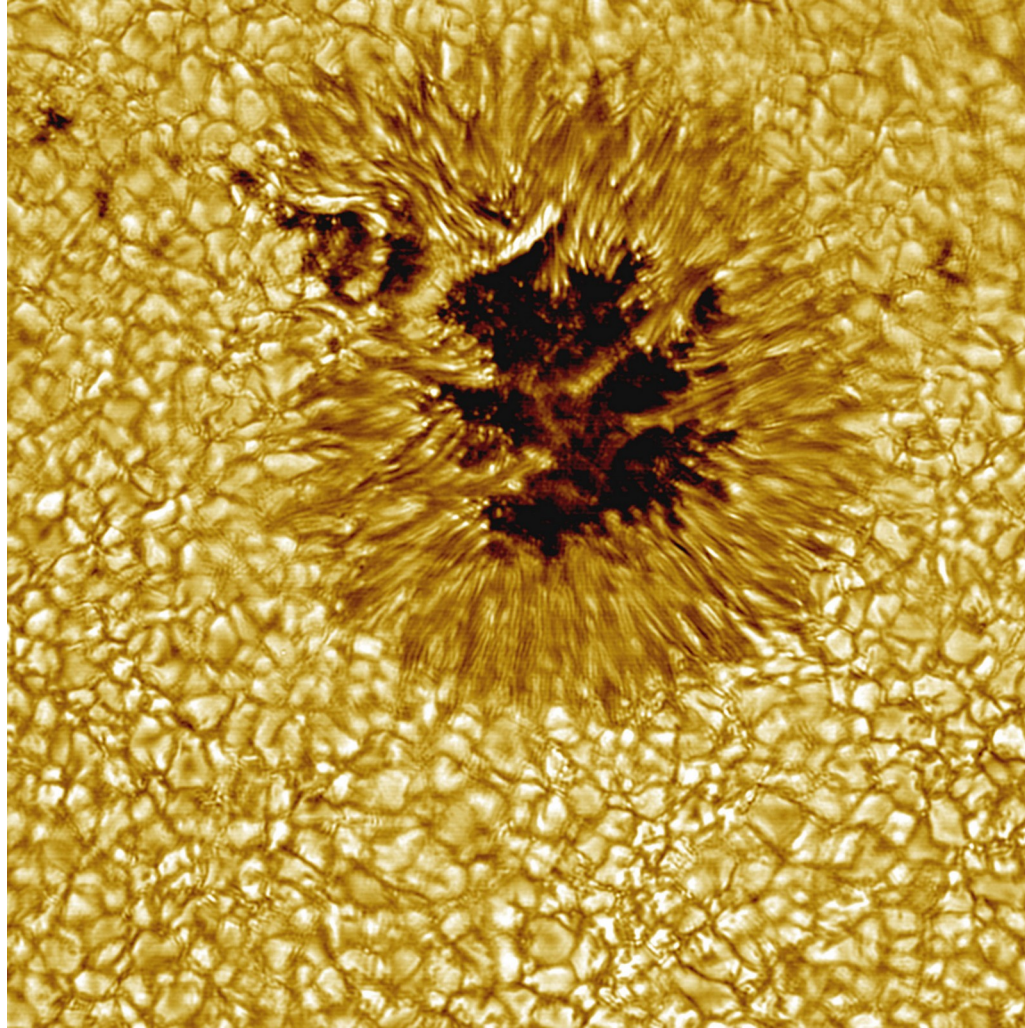
- Sunspots: regions slightly cooler than their surroundings ($T \sim 3800$ K)



(NASA)



Sunspots

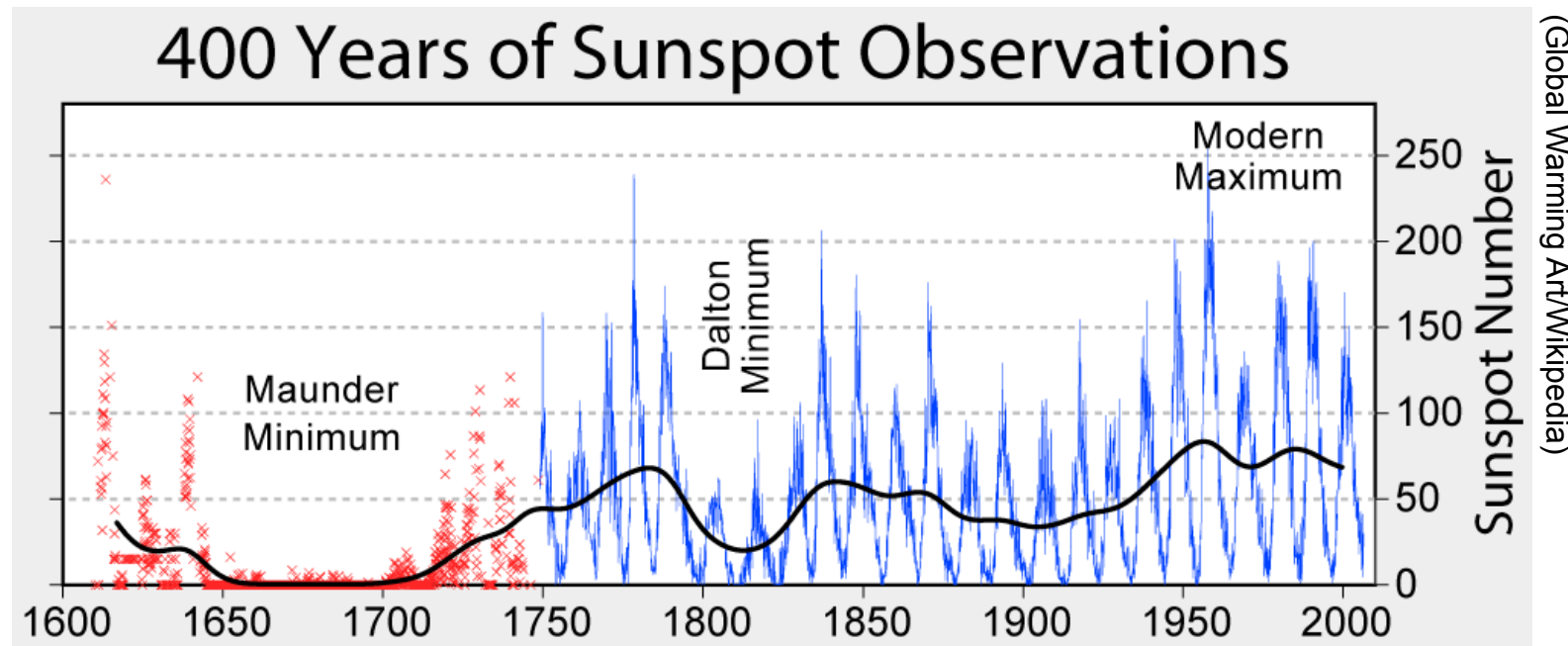


Credit : Vacuum Tower Telescope, NSO, NOAO

PHY521: Stars

Sunspots

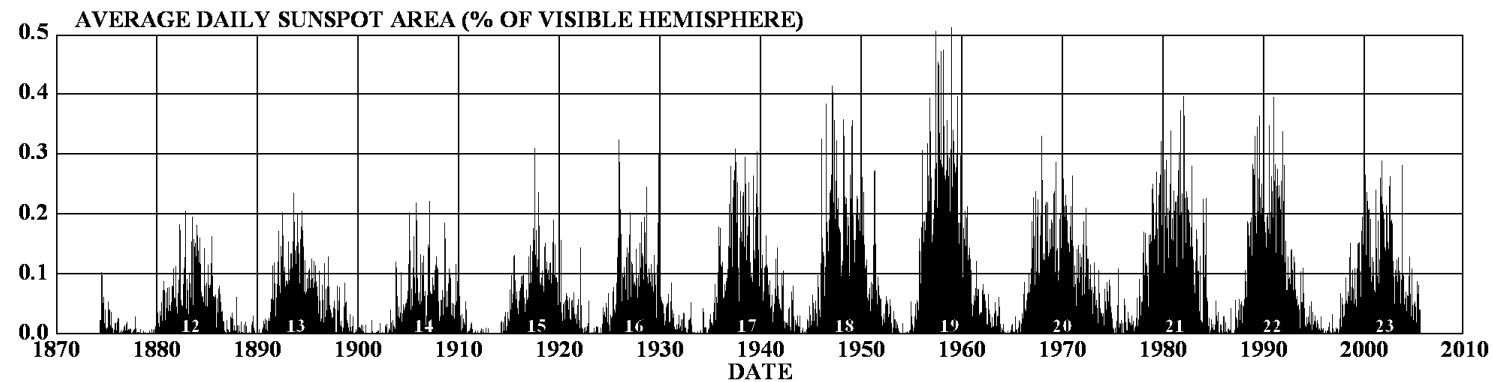
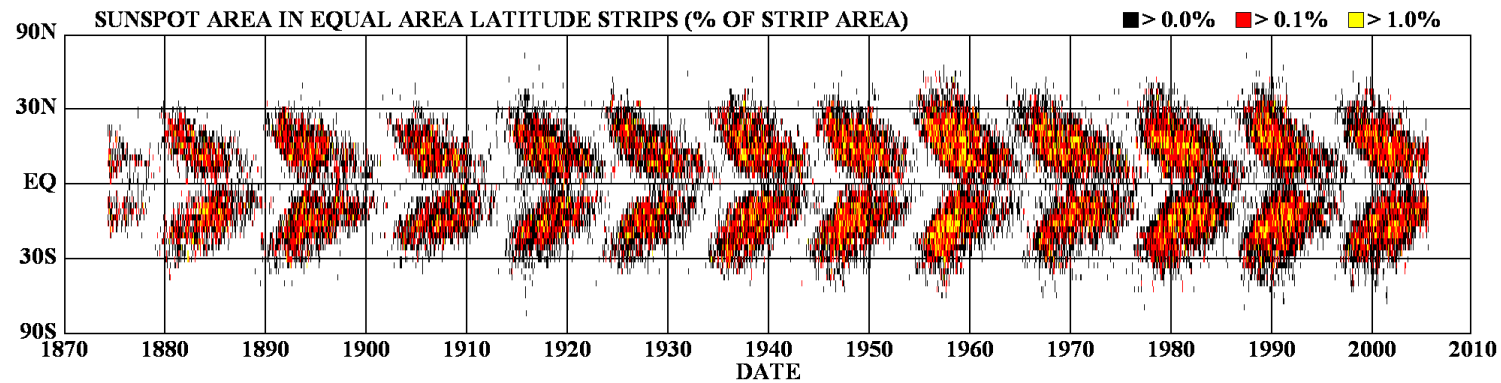
- # varies regularly over 11 year cycle
 - First noticed by E. Walter Maunder (1904)
 - Looking through historical records, very little activity observed from 1645 to 1715—Maunder minimum



Sunspots

- Latitude vs. time: butterfly diagram

DAILY SUNSPOT AREA AVERAGED OVER INDIVIDUAL SOLAR ROTATIONS

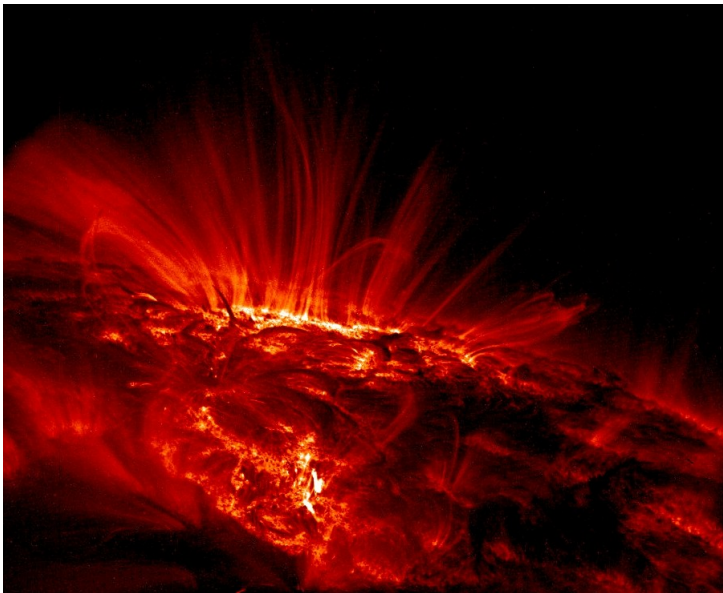


<http://science.msfc.nasa.gov/ssl/pad/solar/images/bfly.gif>

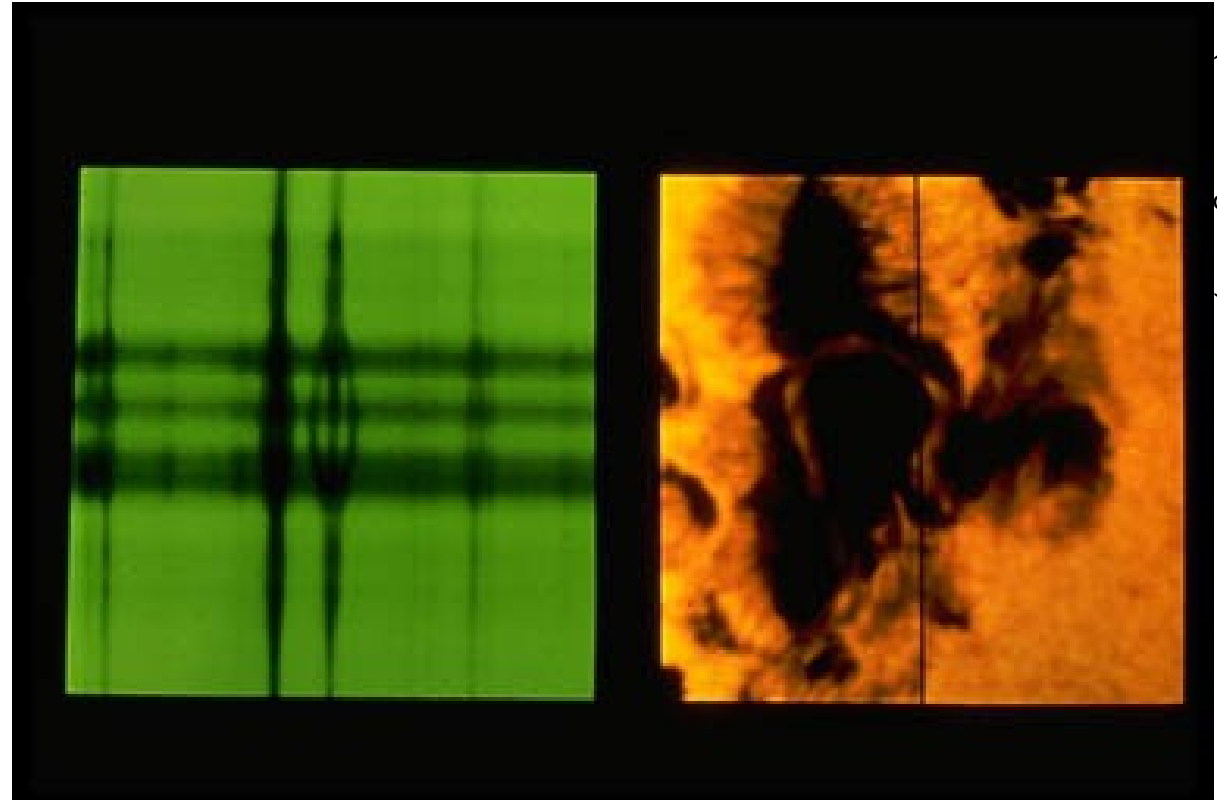
NASA/NSSTC/HATHAWAY 2005/10

Sunspots

- Zeeman effect (splitting of atomic energy levels in B field) allows us to measure magnetic field strength
- Sunspots also appear in pairs



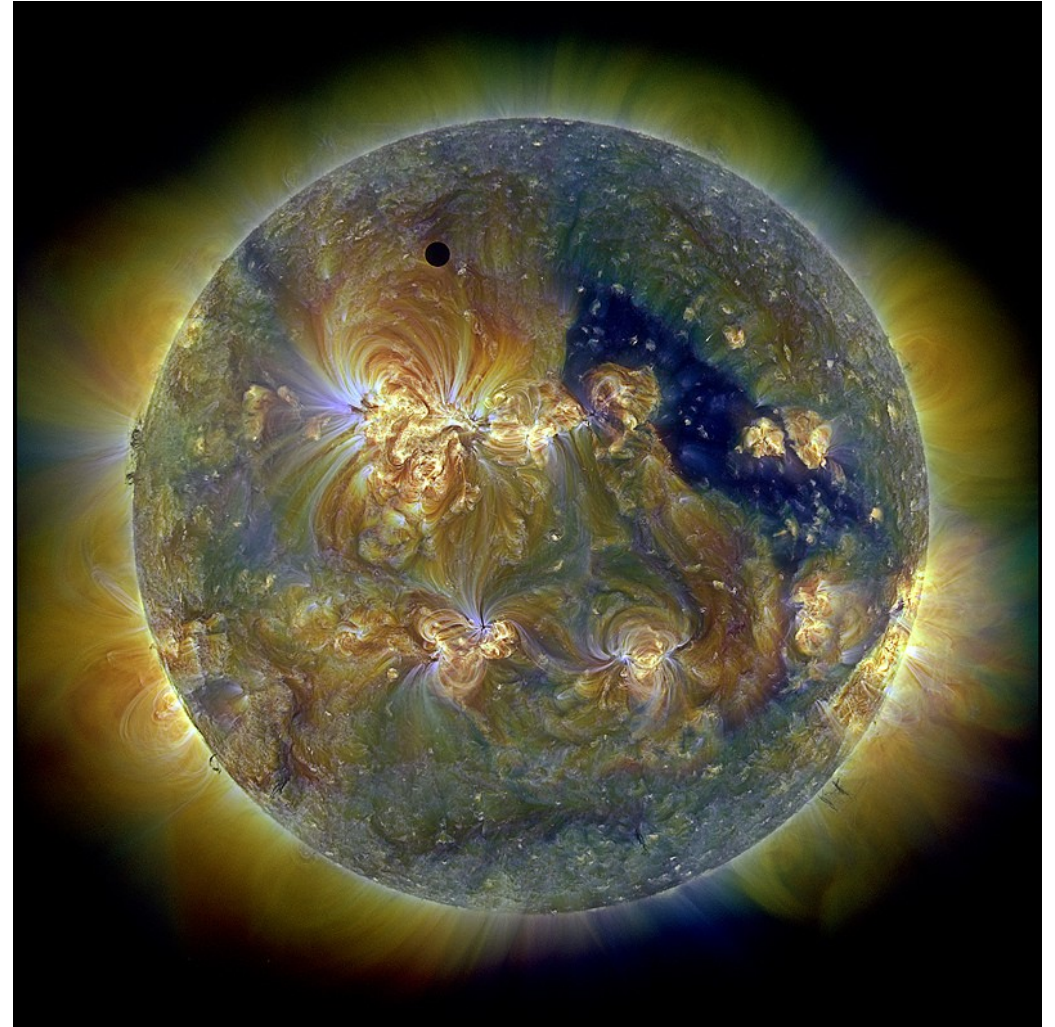
(TRACE)



(K. R. Lang/Tufts)

Sun's Magnetic Field

“An unusual type of solar eclipse occurred last year. Usually it is the Earth's Moon that eclipses the Sun. Last June, most unusually, the planet Venus took a turn. Like a solar eclipse by the Moon, the phase of Venus became a continually thinner crescent as Venus became increasingly better aligned with the Sun. Eventually the alignment became perfect and the phase of Venus dropped to zero. The dark spot of Venus crossed our parent star. The situation could technically be labeled a Venusian annular eclipse with an extraordinarily large ring of fire. Pictured above during the occultation, the Sun was imaged in three colors of ultraviolet light by the Earth-orbiting Solar Dynamics Observatory, with the dark region toward the right corresponding to a coronal hole. Hours later, as Venus continued in its orbit, a slight crescent phase appeared again. The next Venusian solar eclipse will occur in 2117.”



MHD 101

- Stars are a magnetized plasma—the dynamics is described by the equations of magnetohydrodynamics (MHD)
 - Magnetic field enters into the fluid equations
- Ohm's law in a moving magnetized fluid
 - Additional term from the current resulting from Lorentz force:

$$j = \sigma \left(E + \frac{v}{c} \times B \right)$$

- Here σ is the electrical conductivity

- Induction equation:
 - Start with Maxwell's laws

$$\nabla \times B = \frac{4\pi}{c} j + \frac{1}{c} \frac{\partial E}{\partial t}$$

For typical fluid velocities, the displacement current term is negligible

MHD 101

- Together:

$$\nabla \times B = \frac{4\pi}{c} \sigma E + \frac{4\pi}{c} \sigma \frac{v}{c} \times B$$

- Also from Maxwell:

$$-\frac{1}{c} \frac{\partial B}{\partial t} = \nabla \times E = \frac{c}{4\pi\sigma} \nabla \times (\nabla \times B) - \nabla \times \left(\frac{v}{c} \times B \right)$$

- Yields induction equation:

$$\frac{\partial B}{\partial t} = \eta \nabla^2 B + \nabla \times (v \times B)$$

- Resistivity: $\eta = c^2 / (4\pi\sigma)$

MHD 101

- Similar analysis gives the momentum equation as:

$$\frac{\partial v}{\partial t} + (v \cdot \nabla)v = F - \frac{1}{\rho} \nabla \left(P + \frac{B^2}{8\pi} \right) + \frac{(B \cdot \nabla)B}{4\pi}$$

- Note the addition of the **magnetic pressure** term
- For astrophysical flows, generally, the resistive term is small

- Magnetic Reynolds number, compare: $\nabla \times (v \times B)$ to $\eta \nabla^2 B$

$$\mathcal{R}_M \sim \frac{vB/L}{\eta B/L^2} \sim \frac{vL}{\eta} \gg 1$$

- Under these conditions, it can be shown that the **magnetic flux is frozen into the fluid** (see, e.g. Choudhuri)

$$\frac{D}{Dt} \int_S B \cdot dS = 0$$

Magnetic Buoyancy

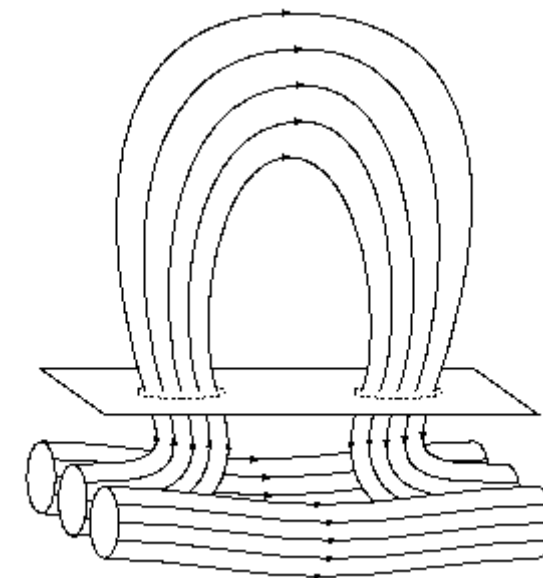
- Important parameter: plasma $\beta = \rho/(B^2/8\pi)$
- Convection bundles up magnetic fields into flux tubes
- Sunspots occur in pairs with opposite polarity
 - Magnetic flux tube breaks out through the surface
 - Sunspots mark the location where the flux tube exits and enters the surface

- Pressure balance

$$P_{\text{out}} = P_{\text{in}} + \frac{B^2}{8\pi}$$

- Magnetic pressure means gas pressure is lower in the flux tube:

$$P_{\text{in}} < P_{\text{out}}$$



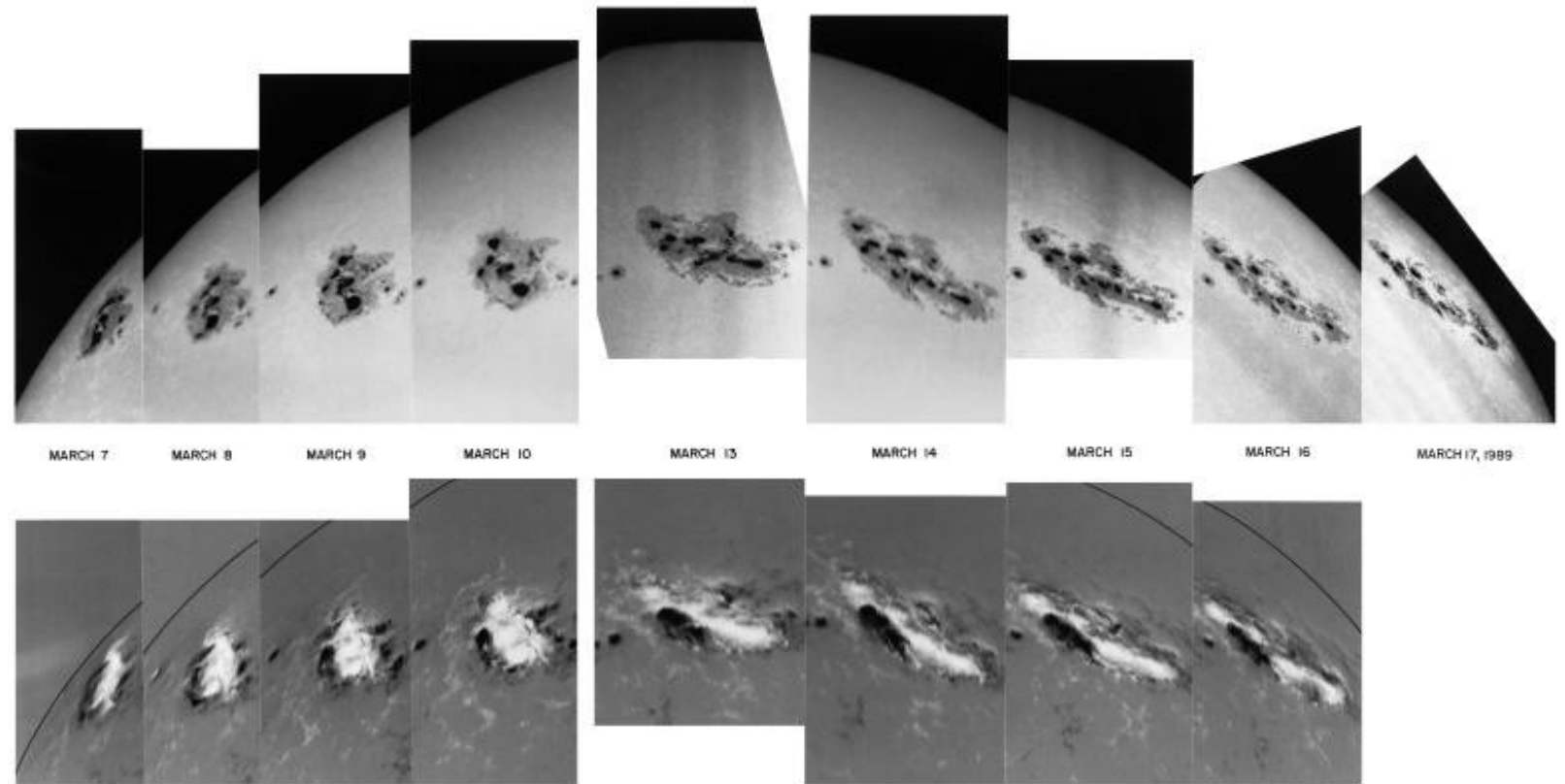
Magnetic buoyancy, attributed to Parker, from <https://www.maths.nottingham.ac.uk/personal/pcm/buoyancy/buoyancy.html>

Magnetic Buoyancy

- Gas pressure difference also implies fluid density inside the flux tube is lower than the external density
 - Flux tube is buoyant
- Temperature is also lower
 - Magnetic pressure in sunspots “replaces” some of the gas pressure
 - Sunspots appear darker than their surroundings

Differential Rotation

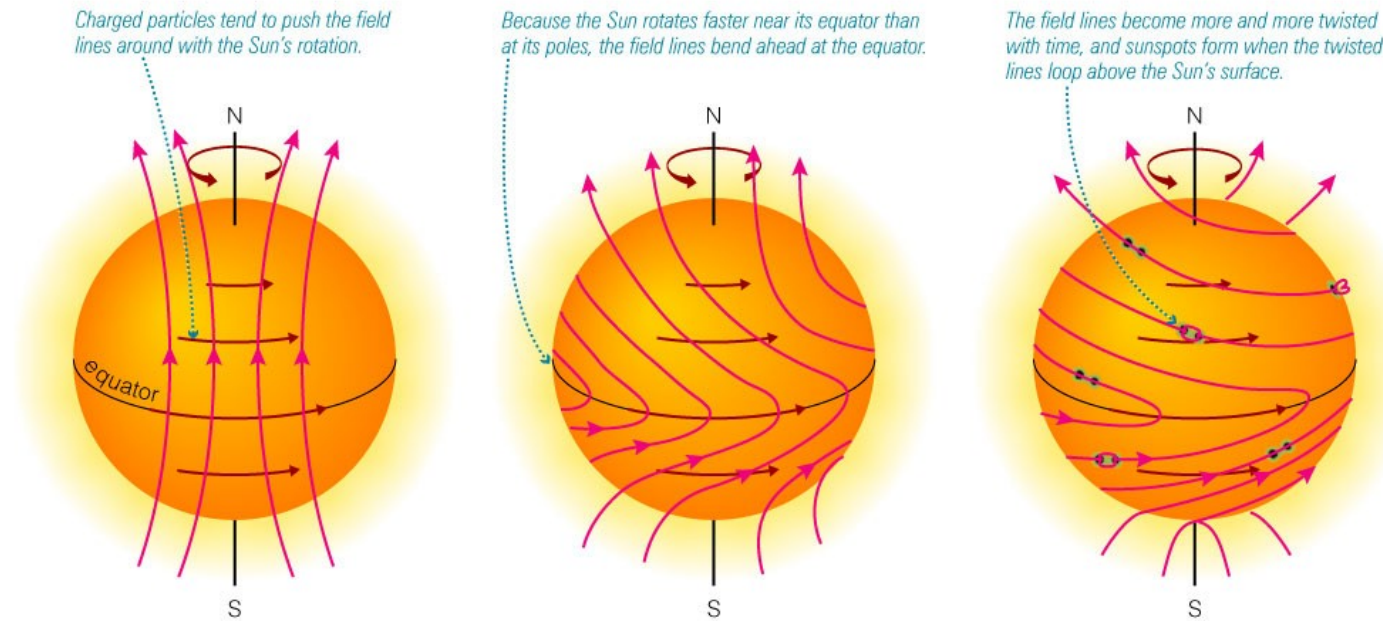
- Sunspots change alignment as they move across the Sun
 - Equator moving faster than poles
 - Differential rotation



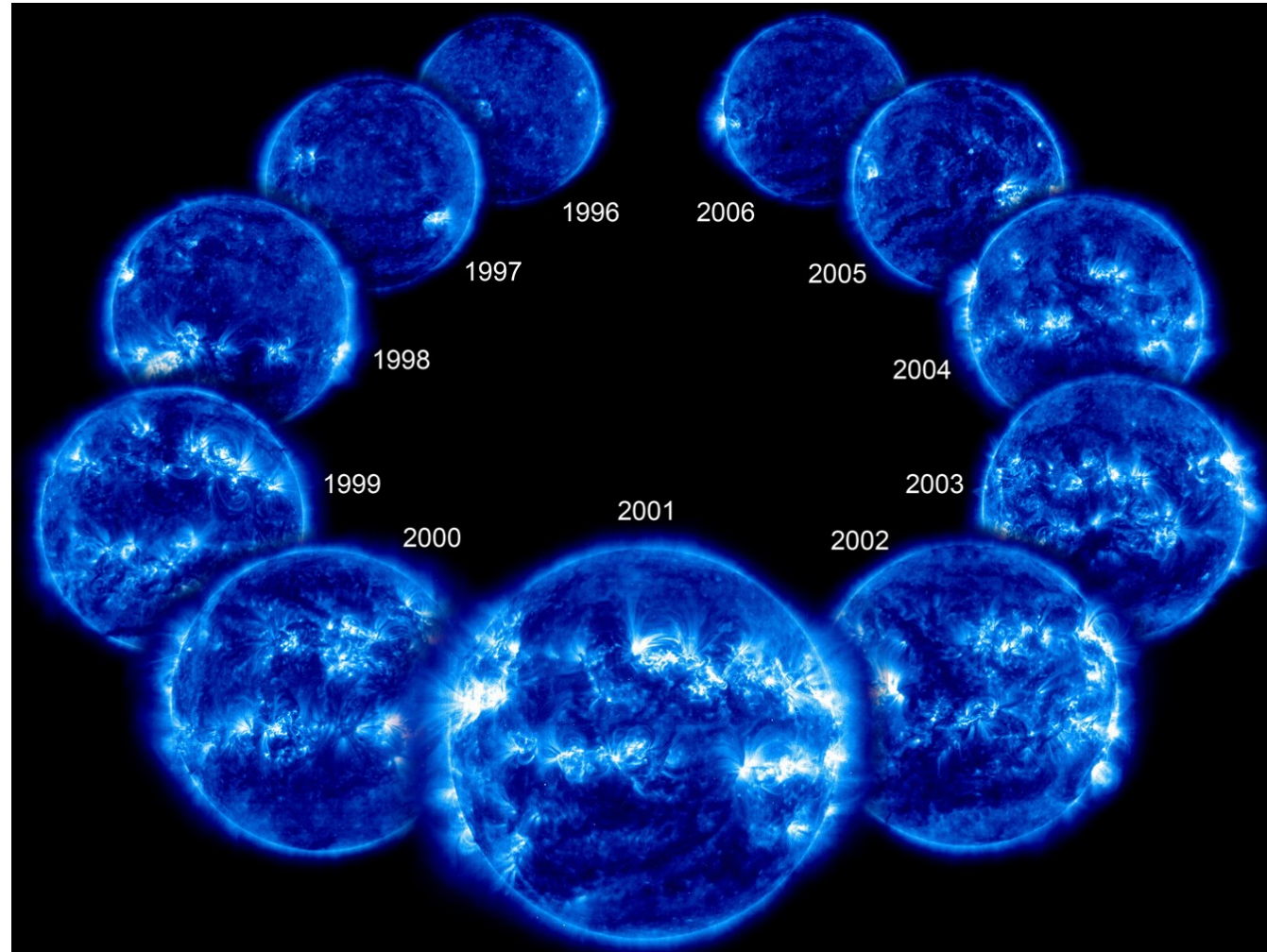
(NSO/AURANSF)

Differential Rotation

- If the Sun's magnetic field was generated in the core, we would expect it to be stable. Earth's magnetic field reverses ~ every million years.
- The Sun's magnetic field is generated close to the surface.
- The magnetic field is dragged with the Sun's differential rotation, and it will wind up.



Magnetic Field Over a Cycle

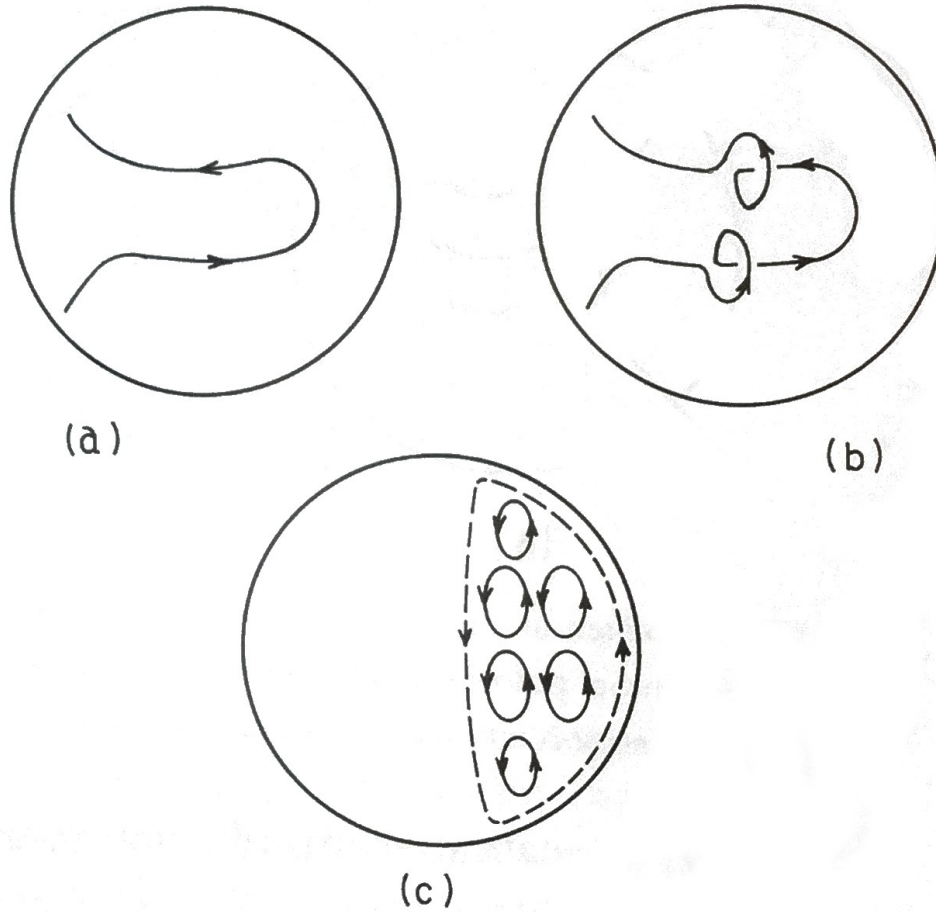


Magnetic Dynamo

- Imagine toroidal (φ direction) and poloidal (in r - θ plane) fields
 - Differential rotation turns poloidal fields into toroidal
 - Something needs to sustain the poloidal field
- Parker: turbulent convective motions stretch the toroidal field up/down (frozen to the fluid) in a rotating body
 - Corkscrew-like loops form
 - These all have the same sense, since the fluid motions and toroidal field both switch from one hemisphere to the other
 - Consequence of vorticity evolution in a rotating frame—see Choudhuri fluids section 9.2.2.
 - Similar to why hurricanes in the Northern / Southern hemisphere have opposite senses
 - Poloidal field amplified by magnetic diffusion of smaller loops in a poloidal plane.

Magnetic Dynamo

Fig. 8.5 Different stages of the dynamo process. See the text for explanations.



(Choudhuri)

Reconnection

- Sometimes the diffusion term cannot be neglected
- Solar flares:
 - Large amount of magnetic energy is released (into heat)
 - $\sim 10^{33}$ erg in minutes
- Consider $\eta \nabla^2 B$
 - Resistivity may be small, but in some regions, grad B can be large
- Ex: current sheet
 - Oppositely directed magnetic fields
 - Magnetic fields decay between sheets via diffusion term
 - Magnetic pressure drops by a large amount
 - Pressure loss: fluid (with B fields) dragged into the center, and more decay
 - Magnetic reconnection event

Reconnection

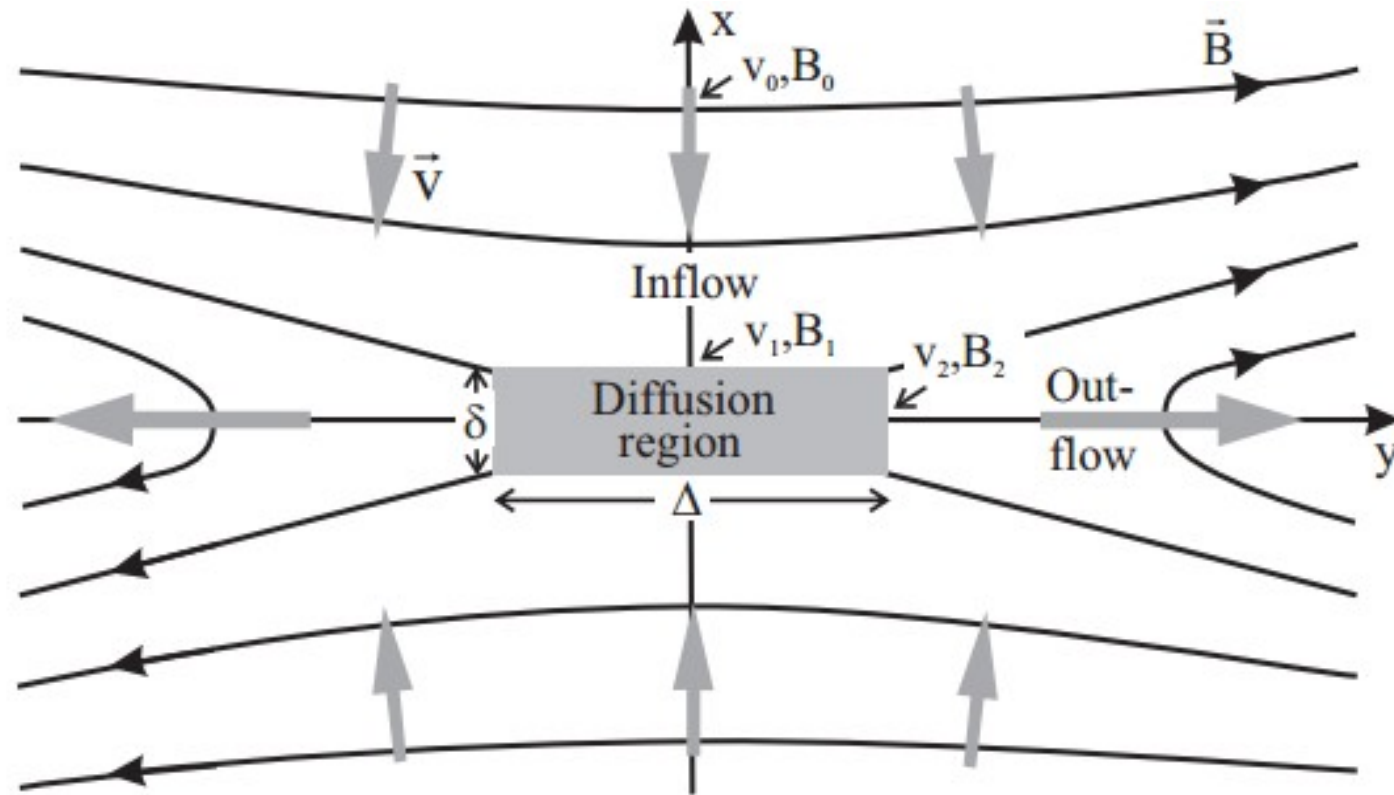
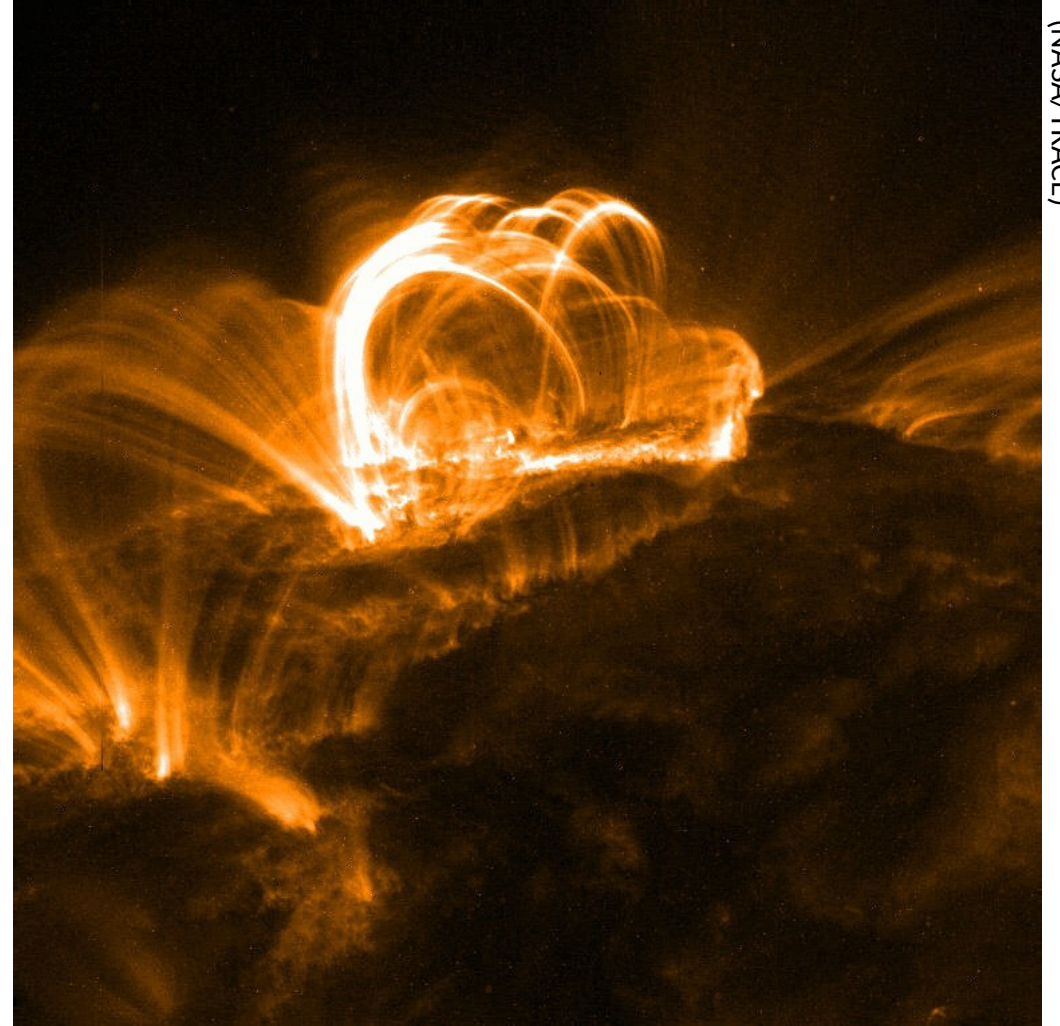


Figure 1. Qualitative pattern of two-dimensional reconnection.

(Encyclopedia of Astronomy and Astrophysics)

Solar Flares

- Solar flares: energetic ejections of particles lasting tens of minutes to hours.
 - Can be seen in H α emission, but radiate across the entire EM spectrum.
 - Not completely understood—ejected particles appear to trace out magnetic field lines, usually associated with sunspots.



(NASA/TRACE)

Solar Flares

- We can make a simple estimate of the energy available in a solar flare
- A large flare has a magnetic field strength of $B \sim 300$ G, and releases 10^{32} erg over an hour
 - Magnetic energy density is:

$$U = \frac{B^2}{8\pi} = \frac{(300 \text{ G})^2}{8\pi} = 3.6 \times 10^3 \text{ erg/cm}^3$$

- Volume of the region associated with the flare is

$$V = \frac{E}{U} = 2.8 \times 10^{28} \text{ cm}^3$$

- Or a lengthscale of

$$l = V^{1/3} = 30000 \text{ km}$$

Solar Flares

- Just like soundwaves communicate a disturbance in a fluid, Alfven waves communicate disturbances along magnetic fields

- Alfven velocity is:

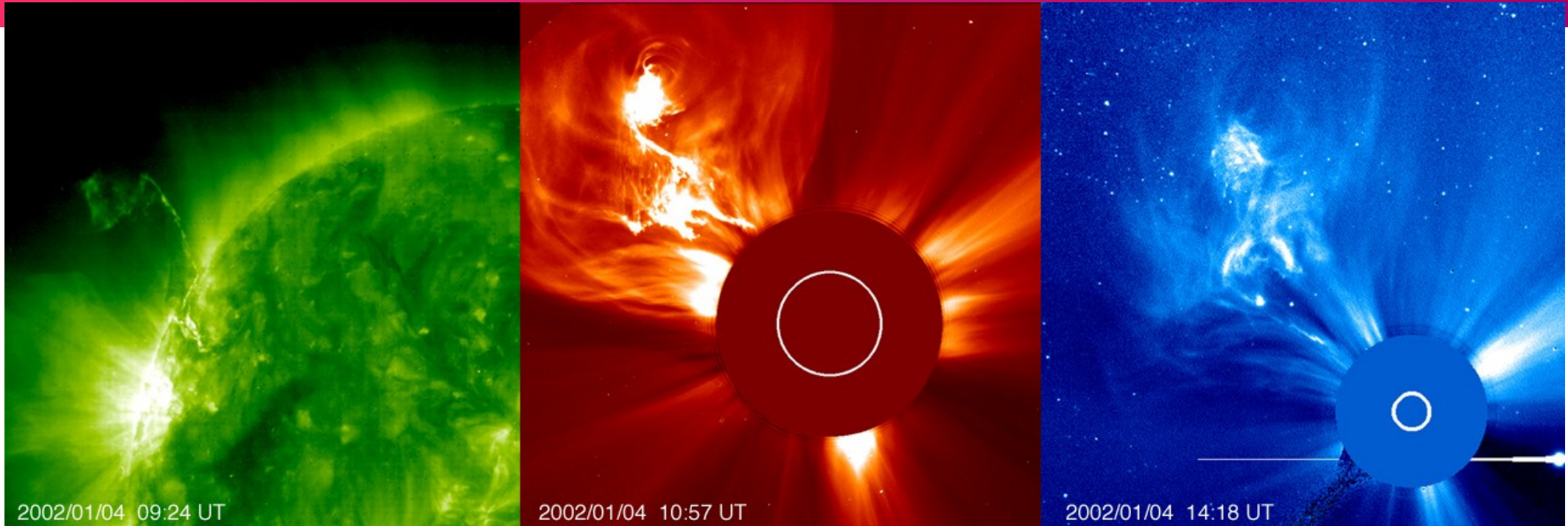
$$v_A = \frac{B}{\sqrt{4\pi\rho}} = 1.2 \times 10^6 \text{ cm/s}$$

- And the timescale to traverse the region is:

$$t \sim \frac{l}{v_A} = 2500 \text{ s} = 0.7 \text{ hr}$$

- This timescale is consistent with what we stated up front, and suggests that the idea of flares being magnetic in origin is valid.

Coronal Mass Ejection



Caption: Another spectacular Coronal Mass Ejection (CME) took off from the Sun in the early hours of January 4, starting off as a filament eruption seen by the Extreme ultraviolet Imaging Telescope (EIT) in the 195 Å images. The complexity and structure of the CME as it passed through the Large Angle and Spectrometric Coronagraph (LASCO) C2 and C3 fields of view amazed even experienced solar physicists at the SOHO operations center.

Although the center of the CME was directed almost a full 90 degrees away from the Sun-Earth connection line, it still appeared as a (weak) full halo event, opening up the possibility of a small impact on Earth's space weather. However, no clear effect was recorded.

http://soho.nascom.nasa.gov/hotshots/2002_01_04/

Coronal Heating

- The solar corona is \sim million K—why?
- Corona: low β —magnetic fields dominate dynamics
- Photospheric motions drag field lines around and this motion in the corona, tangling field in the corona
 - Current sheets form
 - Magnetic energy dissipated heats corona
- Still very controversial and not well understood

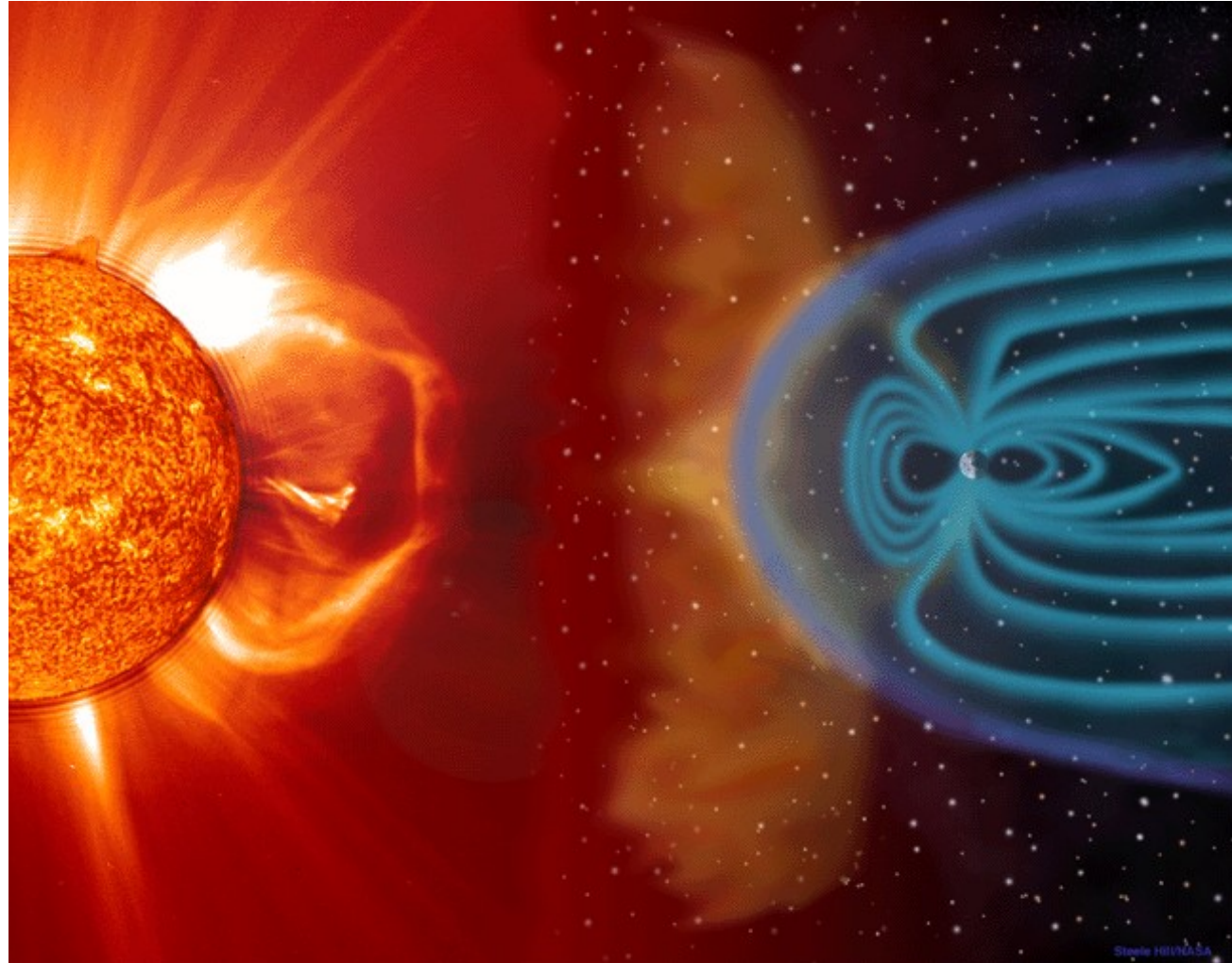
Magnetic Braking

(Choudhuri)

- The Sun rotates slower than we would expect from considering conservation of angular momentum as a gas cloud collapses
- During star formation:
 - Magnetic field lines are frozen into the fluid—they want to force the star to rotate as a solid body (resist tangling)
 - Coupling of magnetic field in the star to the surrounding plasma
 - Star wants to spin faster than surrounding medium—coupling of magnetic field resists this.
 - Angular momentum transported outward (this further helps the collapse)
- The solar wind interacting with the Sun's magnetic field can also remove angular momentum

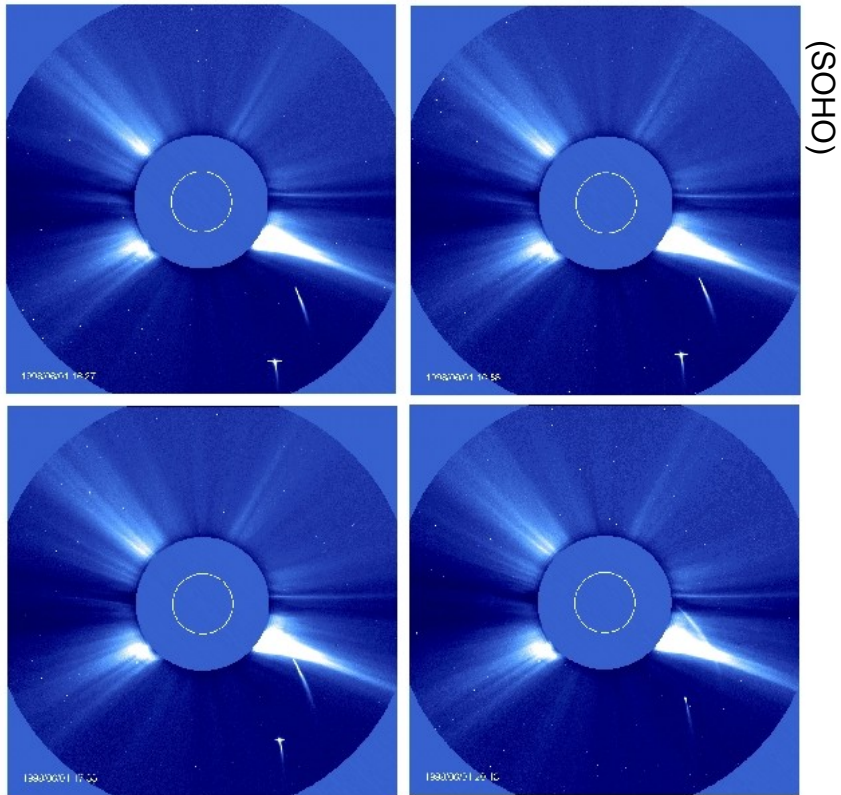
Solar Wind

- Mass loss rate of $\sim 10^{-14} M_{\odot}/\text{yr}$



Solar Wind

- Solar wind + atmosphere = aurora



Why a Solar Wind

- We can make a simple estimate of why there is a solar wind (this follows C&O section 11.2, a slightly better argument is in Choudhuri fluids, section 4.4)
- Assume that we are isothermal (this is not a bad approximation)

- HSE:

$$\frac{dP}{dr} = -\frac{GM_{\odot}\rho}{r^2}$$

- Completely ionized H:

$$n_H \sim \frac{\rho}{m_p} \quad P = 2n_H k_B T$$

- Together:

$$\frac{d}{dr}(2n_H k_B T) = -\frac{GM_{\odot}n_H m_H}{r^2}$$

- Which has the solution:

$$n(r) = n_0 e^{-\lambda(1-r_0/r)}$$

- With:

$$\lambda \equiv \frac{GM_{\odot}m_p}{2k_B T r_0}$$

Why a Solar Wind

- We can write the solution as:

$$P = P_0 e^{-\lambda(1-r_0/r)}$$

- First issue: $P(r \rightarrow \infty) \neq 0$
 - If you put in reasonable #s for the corona, you find that the asymptotic pressure is far greater than you would expect for the interstellar medium
- We conclude that the corona is not in HSE!

Parker Solar Wind Problem

- Parker (1958) predicted the properties of the solar wind from a steady-state model for it
 - He argued that it is supersonic at large distances from the Sun

- Consider continuity and momentum equation:

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho u) = 0$$
$$\frac{\partial(\rho u)}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho u^2) + \frac{\partial p}{\partial r} = -\frac{GM_{\odot}}{r^2} \rho$$

- Assume an isothermal gas:

$$p = \rho c^2$$

Parker Solar Wind Problem

- Steady state (no time dependence):

$$\frac{1}{r^2} \frac{d}{dr} (r^2 \rho u) = 0$$

$$\frac{1}{r^2} \frac{d}{dr} (r^2 \rho u^2) + \frac{dp}{dr} = -\frac{GM_{\odot}}{r^2} \rho$$

- Combining:

$$u \frac{du}{dr} + \frac{1}{\rho} \frac{dp}{dr} = -\frac{GM_{\odot}}{r}$$

- With EOS:

$$u \frac{du}{dr} + \frac{c^2}{\rho} \frac{d\rho}{dr} = -\frac{GM_{\odot}}{r}$$

- And continuity:

$$u \frac{du}{dr} + c^2 \left(-\frac{2}{r} - \frac{1}{u} \frac{du}{dr} \right) = -\frac{GM_{\odot}}{r^2}$$

Parker Solar Wind Problem

- Rewriting:

$$\frac{1}{2} \left(1 - \frac{c^2}{u^2} \right) \frac{du^2}{dr} = -\frac{GM_{\odot}}{r^2} + \frac{2c^2}{r}$$

- Critical sonic point ($u = c$) at:

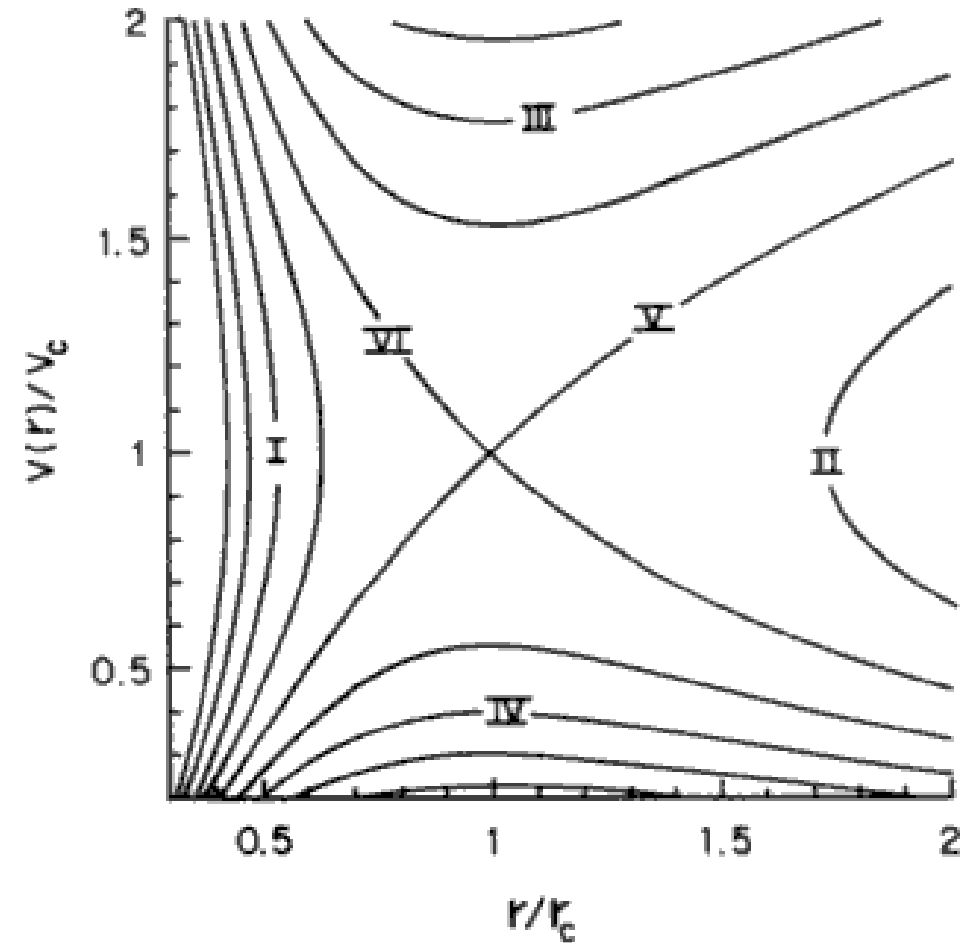
$$r_c = \frac{GM_{\odot}}{2c^2}$$

- Non-dimensional: $\xi = r/r_c$

$$\left(1 - \frac{1}{M^2} \right) \frac{dM^2}{d\xi} = -\frac{4}{\xi^2} + \frac{4}{\xi}$$

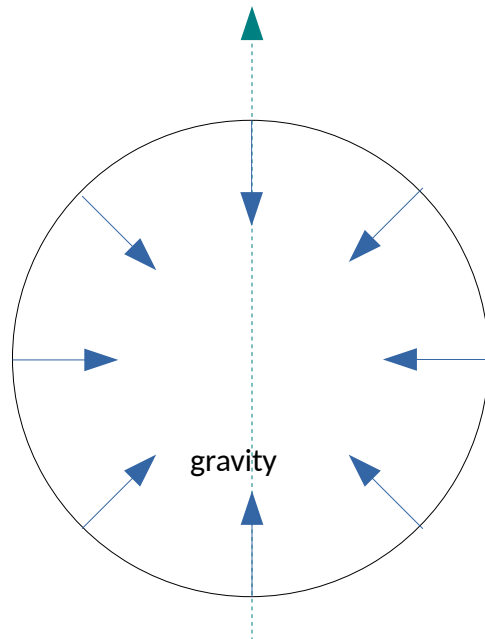
Parker Solar Wind Problem

- Solution cases
- If we pass through $\xi = 1$, then the RHS is 0
 - $M = 1$ (transonic) or
 - $dM^2/d\xi = 0$
- Observations show that the transonic solution is correct
- Interesting note: run in reverse, this problem also describes Bondi accretion

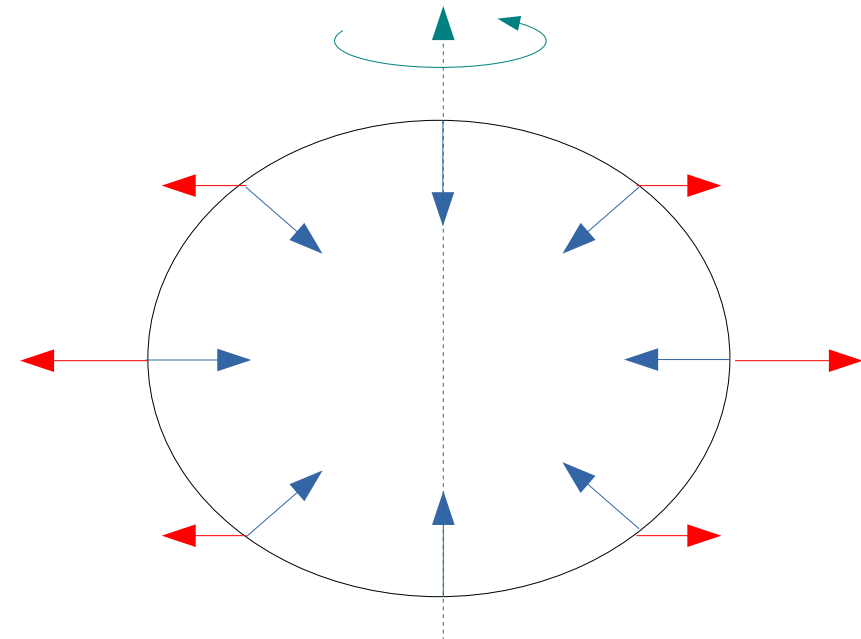


More on Rotation

- Rotating self-gravitating bodies are no longer spherical
 - Centrifugal force will lessen the effect of gravity on the equator



No rotation



rotation

Break-up!

- Point on surface of star

- Balance:

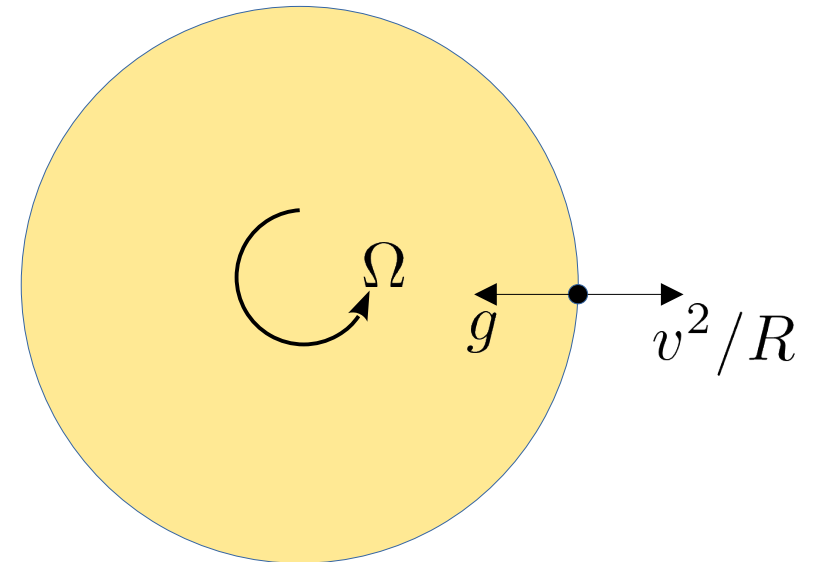
$$\frac{v^2}{R} = \Omega^2 R = \frac{GM}{R^2}$$

- Critical angular velocity:

$$\Omega_0 = \left(\frac{GM}{R^3} \right)^{1/2}$$

- Break-up for $\Omega > \Omega_0$

- For Sun: $\Omega_0 = 0.12$ days (much faster than Sun's rotation)



More on Rotation

- We measure rotation rates in terms of the critical rate for breakup (compare gravity to centrifugal force)
- Sun rotates slowly
 - More massive stars tend to rotate faster (again mass cut is around 1.5 solar masses: outer convective region)
- When does solar wind decouple from stellar interior's rotation?
 - Massive stars: near photosphere
 - Low mass: much further out
 - Magnetic field frozen into star, and it rotates with the star
 - Co-rotating wind gains angular momentum as it moves outward—interior loses it
- Early sun must have rotated much more quickly
 - We didn't include that in our evolutionary calculations...

More on Rotation

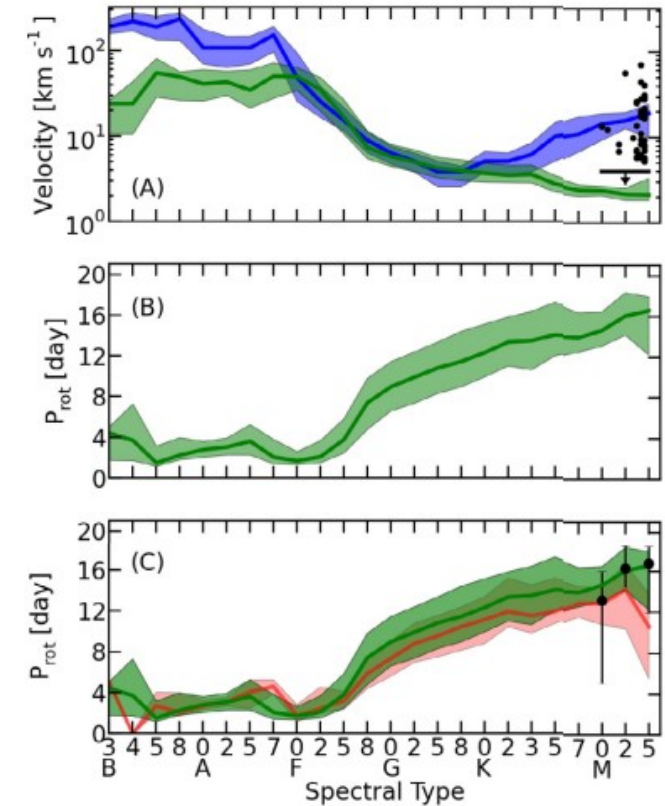


Fig. 2. Panel **a**) the blue curve is the median equatorial velocity $(4/\pi) \langle v \sin i \rangle$ for each spectral type from [Głębocki & Gnaniński \(2005\)](#). The green curve shows the equatorial rotation velocity of the *Kepler* targets, $\bar{v}(s.t.)$, derived from the measured rotation periods and the KIC radii. The black points show measurements by [Reiners et al. \(2012\)](#). In this sample 201 stars have an upper $v \sin i$ limit of 4 km s^{-1} (due to instrumental limitations), these stars are represented by the solid bar. Panel **b**) rotation periods P_{rot} of the stars in our sample, averaged within each spectral type. Panel **c**) same as panel **b**), but for comparison we show the median of the rotation periods measured by [McQuillan et al. \(2013\)](#) (black points with errorbars), for the stars overlapping with our sample. Similarly, the red curve shows the median of the rotation periods found by [Debosscher et al. \(2011\)](#). Shaded areas and error bars span the upper and lower 34th percentile values from the median.

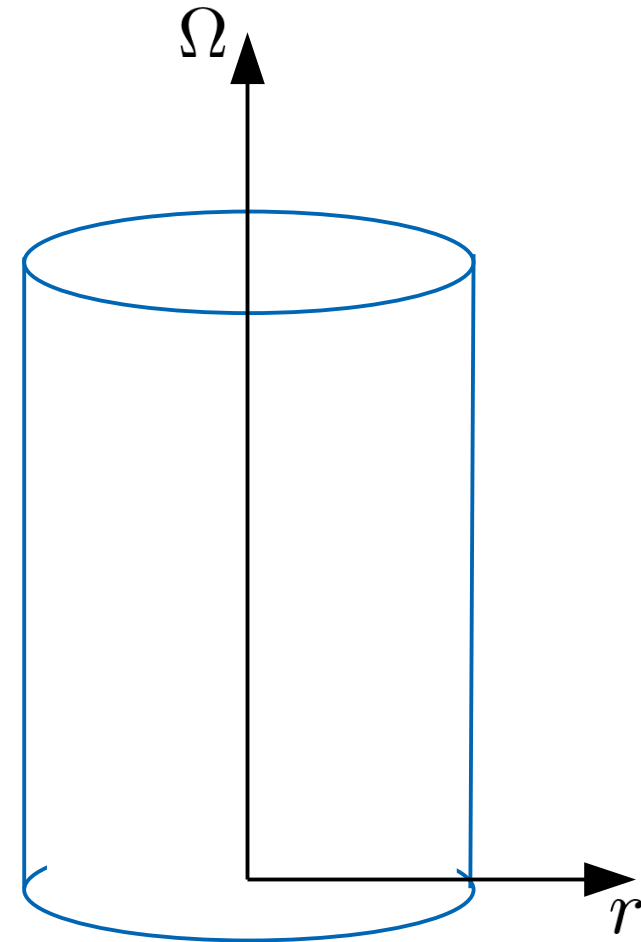
Centrifugal Potential

- Consider cylindrical coords with rotation axis in z-direction
 - We can do this without loss of generality
- Our centrifugal term is:

$$-\Omega \times (\Omega \times \mathbf{r})$$

- \mathbf{r} is the distance from the rotation axis
 - Any component in the same direction as Ω vanishes in the cross product
- We see that this is identical to:

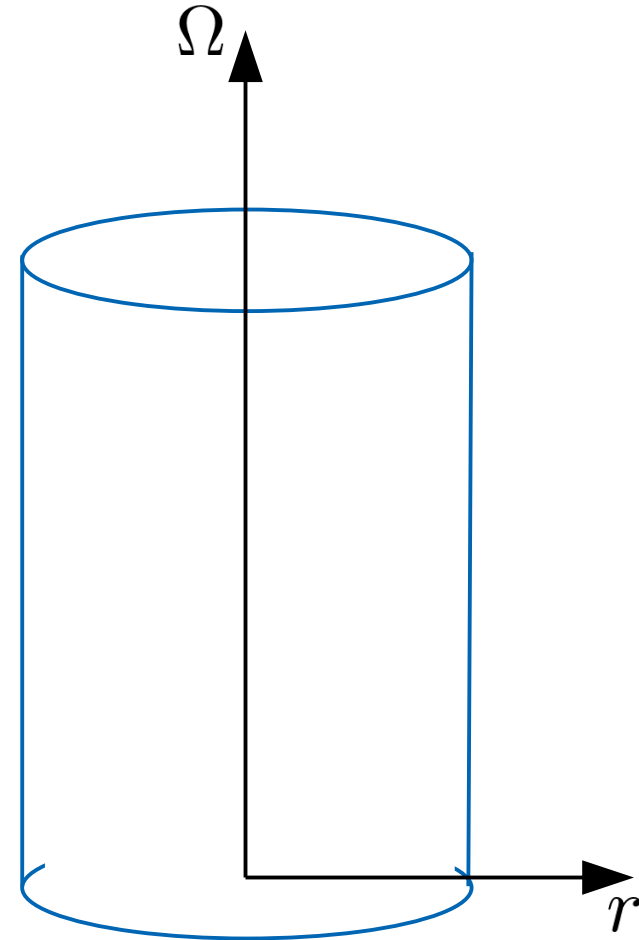
$$\frac{1}{2} \nabla (|\Omega \times \mathbf{r}|^2)$$



Centrifugal Potential

- We want to write the force as: $\mathbf{g}_{\text{eff}} = -\nabla\Phi$
- Effective potential:

$$\Phi_{\text{eff}} = \Phi - \frac{1}{2}|\boldsymbol{\Omega} \times \mathbf{r}|^2$$



Effects of Rotation

- Consider a star that is:
 - Homogeneous composition
 - In thermal balance + radiation transfer
 - Hydrostatic
 - Rotates as a solid ($\Omega = \text{constant}$)

- HSE is then:

$$\frac{\nabla p}{\rho} = -\nabla\Phi_{\text{eff}}$$

- P is constant on equipotentials

- Taking curl:

$$\nabla p \times \nabla \rho = 0$$

- Density is also constant on equipotentials

- If μ is constant, then T is constant on equipotentials too (for ideal gas)
- But g_{eff} is not constant

Effects of Rotation

- Thermal balance:

$$\nabla \cdot \mathcal{F} = \rho\epsilon$$

- where

$$\mathcal{F} = \frac{L}{4\pi r^2}$$

- This is equivalent to our thermal equilibrium expression (in spherical coords)

$$\frac{dL}{dr} = 4\pi r^2 \rho\epsilon$$

- For radiation, we know

$$\begin{aligned} L &= -4\pi r^2 \frac{c}{3\kappa\rho} \nabla(aT^4) \\ &= -4\pi r^2 \frac{ac}{3\kappa\rho} 4T^3 \nabla T \end{aligned}$$

- Now, $T = T(\Phi_{\text{eff}})$, so

$$\nabla T = \frac{dT}{d\Phi_{\text{eff}}} \nabla \Phi_{\text{eff}}$$

- This gives:

$$\mathcal{F} = \frac{L}{4\pi r^2} = -\frac{4ac}{3} \frac{T^3}{\kappa\rho} \frac{dT}{d\Phi_{\text{eff}}} \nabla \Phi_{\text{eff}}$$

Effects of Rotation

- Notice that ϵ and κ are functions of ρ and T , therefore they are constant on equipotentials

- Rewriting:

$$\mathcal{F} = f(\Phi_{\text{eff}})\nabla\Phi_{\text{eff}} = -f(\Phi_{\text{eff}})g_{\text{eff}}$$

- Divergence:

$$\nabla \cdot \mathcal{F} = \frac{df}{d\Phi_{\text{eff}}} \left(\frac{d\Phi_{\text{eff}}}{dn} \right)^2 + f\nabla^2\Phi_{\text{eff}} = \rho\epsilon$$

- d/dn is the normal direction, but

$$\left| \frac{d\Phi_{\text{eff}}}{dn} \right| = |g_{\text{eff}}|$$

- Poisson:

$$\nabla^2\Phi_{\text{eff}} = \nabla^2\Phi_{\text{grav}} - \frac{1}{2}\nabla^2\Omega^2 r^2$$

- Putting it all together, we find:

$$\frac{df}{d\Phi_{\text{eff}}}g_{\text{eff}}^2 + f[4\pi G\rho - 2\Omega^2] = \rho\epsilon$$

Effects of Rotation

- Here's the issue:

$$\frac{df}{d\Phi_{\text{eff}}} g_{\text{eff}}^2 + f [4\pi G\rho - 2\Omega^2] = \rho\epsilon$$

- g_{eff} is not constant on level surfaces but the RHS of this expression is
- To make this work, we require $df/d\Phi_{\text{eff}} = 0$
- But this then implies:

$$\epsilon \propto 1 - \frac{\Omega^2}{2\pi G\rho}$$

- How can a microscopic property (energy generation) depend on a macroscopic property (rotation)?
- This is the von Zeipel paradox
- To break this dependence, we need to relax one of our assumptions...
 - Uniform rotation \rightarrow differential rotation?
 - Non-zero flux divergence away from core (energy generation region)?
 - e.g. flow can transport some energy so no longer in pure HSE

Eddington-Sweet Circulation

- Eddington-Sweet: meridional circulation can transport energy and resolve von Zeipel's paradox

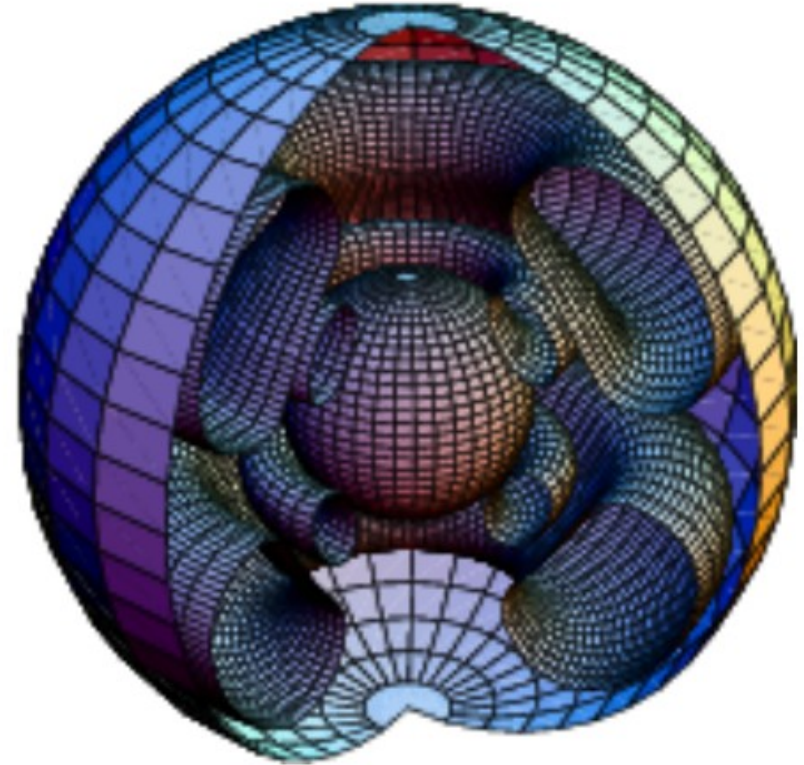


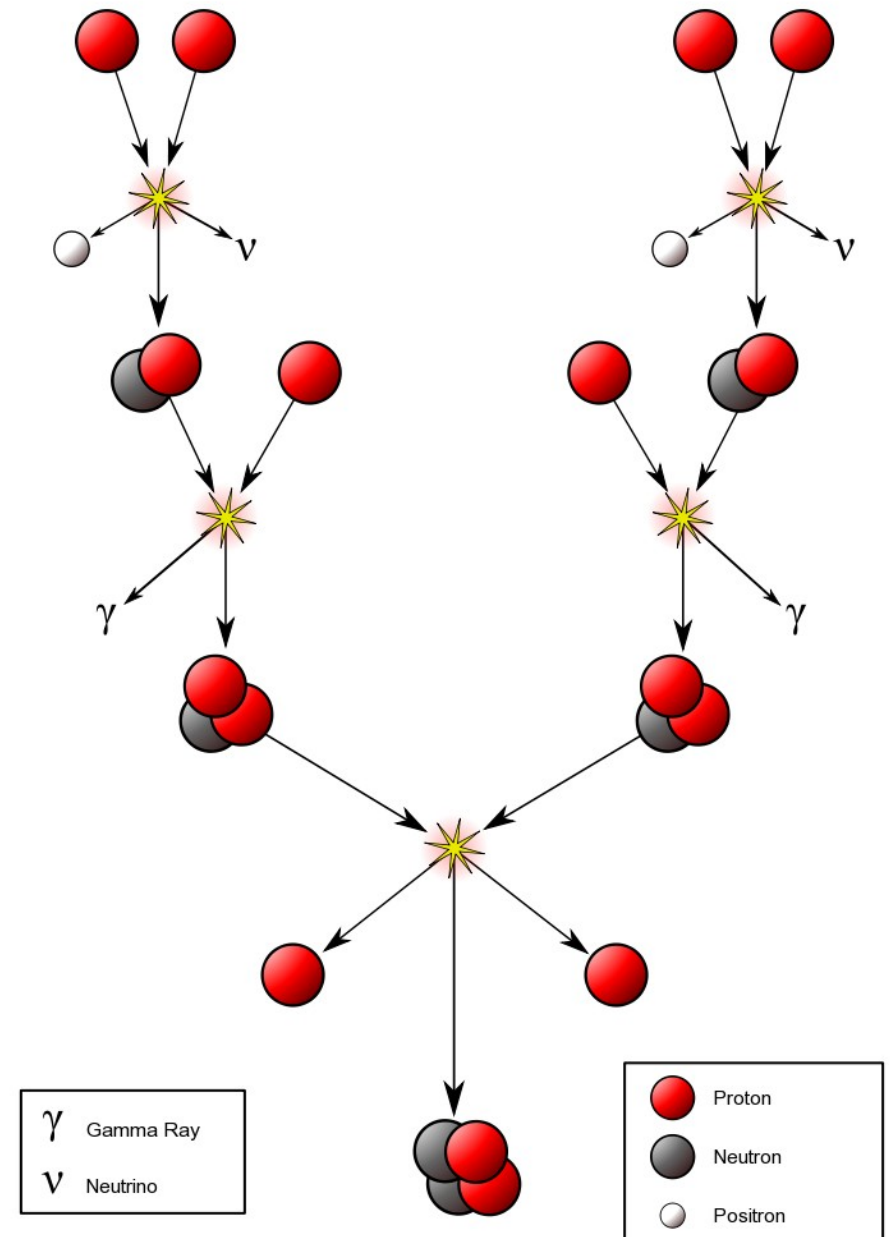
Fig. 1 Streamlines of meridional circulation in a rotating $20 M_{\odot}$ model with solar metallicity and $v_{\text{ini}} = 300 \text{ km s}^{-1}$ at the beginning of the H-burning phase. The streamlines are in the meridian plane. In the upper hemisphere on the right section, matter is turning counterclockwise along the outer streamline and clockwise along the inner one. The outer sphere is the star surface and has a radius equal to $5.2 R_{\odot}$. The inner sphere is the outer boundary of the convective core. It has a radius of $1.7 R_{\odot}$ (Illustration taken from Meynet & Maeder 2002).

How Can We Confirm Stellar Models?

- Solving the equations of stellar structure can match observations
- But also:
 - Neutrinos
 - Helioseismology

Solar Neutrinos

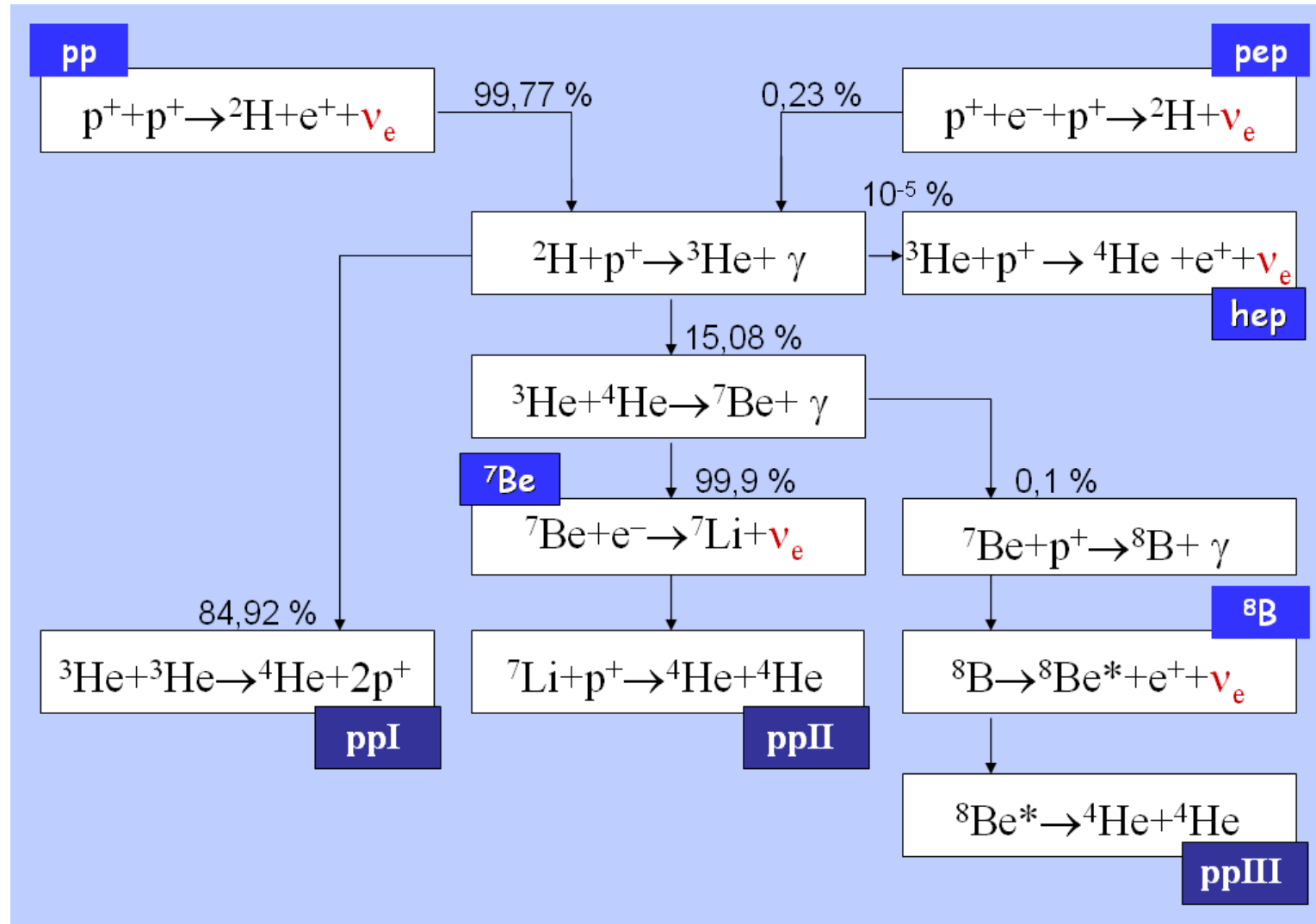
- $4\ ^1\text{H} \rightarrow 1\ ^4\text{He}$ releases 2 neutrinos.
- Neutrinos interact weakly with matter—free stream out of the Sun.
- We can:
 - Predict the # of neutrinos we expect to be produced in the Sun
 - Create detectors in hopes of observing them
- If we see the amount we predict, then things are good!



Solar Neutrinos

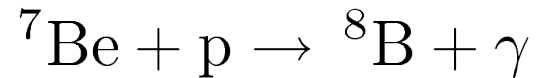
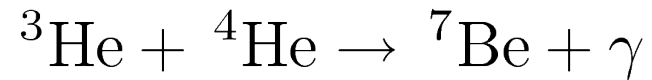
- Neutrinos are hard to detect—they pass right through the Sun, why can we see stop them?
- $\sim 10^{10}$ neutrinos pass through every cm^2 of your body per sec.
- To stop the average neutrino, you would need 1 lightyear of lead!
 - This is a statement of the mean free path
- Large detector → hope for a few chance encounters with solar neutrinos.
 - Usually, these detectors are placed in deep mines to shield them from background radiation.

Solar Neutrinos



Solar Neutrinos

- The neutrinos produced in the first reaction of the p-p chain are of pretty low energy.
- Other branches of the p-p chain produce higher energy neutrinos. Once ${}^3\text{He}$ is made, it could react via:



- When this boron nucleus decays, a neutrino is produced



Solar Neutrinos

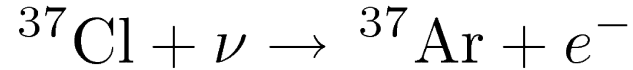
- Neutrino produced from the ${}^8\text{B}$ decay is of high enough energy that we have a chance to detect it.
- **Davis Experiment** was built in the 1960s in the Homestake mine in South Dakota, 1500 m below ground.
- It contained 400,000 L of dry cleaning fluid (C_2Cl_4).



(DOE)

Solar Neutrinos

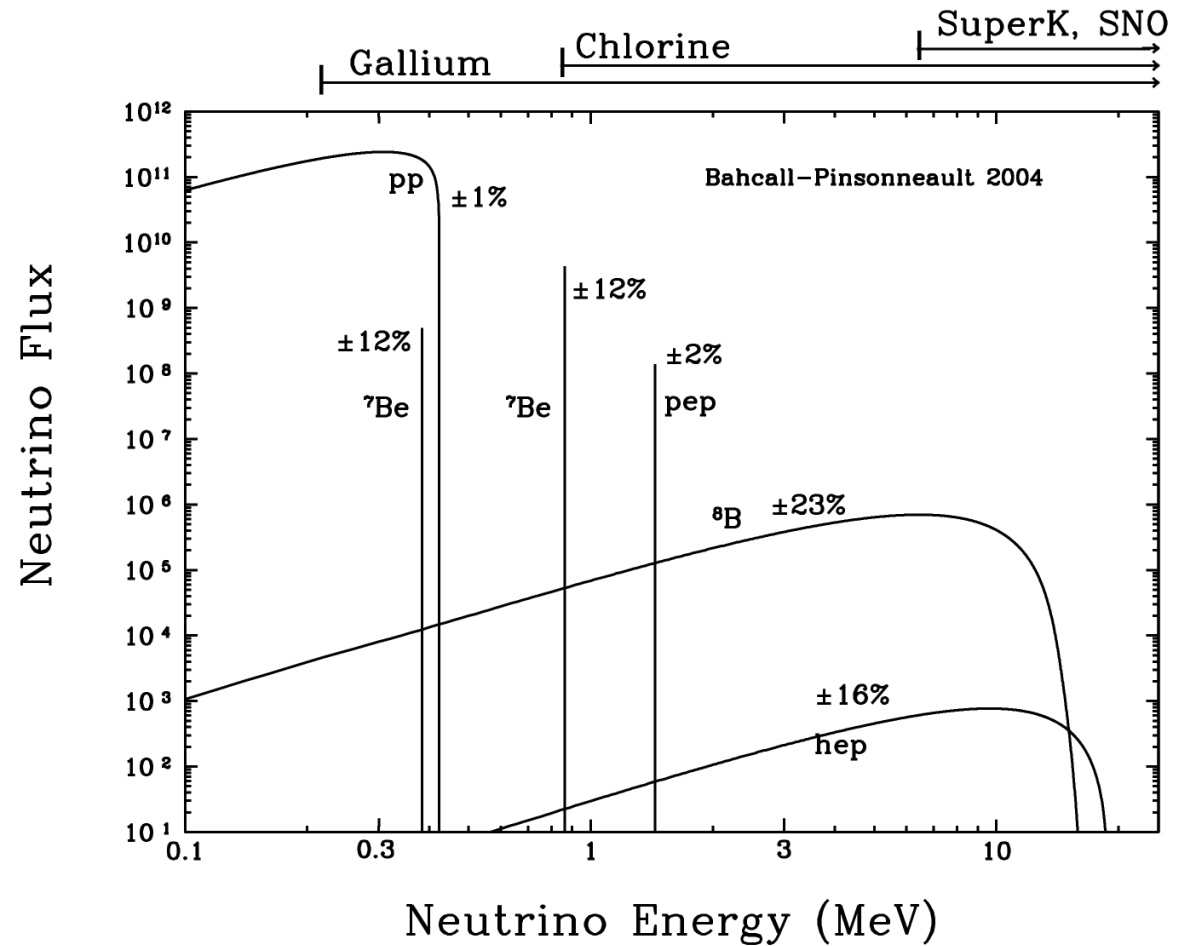
- Detector filled with chlorine-37. Looks for:



- Every few weeks, the tank is drained and the Argon inside is measured.
- In 2 days, about 10^{22} neutrinos will pass through the tank and measurements showed that only one interacts.
- After many decades of measuring, the experiment found only 1/3rd of the expected neutrinos—[solar neutrino problem](#).

Solar Neutrinos

- Other reactions produce neutrinos with different energies.
- Detectors with gallium instead of chlorine are sensitive to neutrinos from a wider range of reactions.
- These experiments still measured only 1/3rd of the expected #

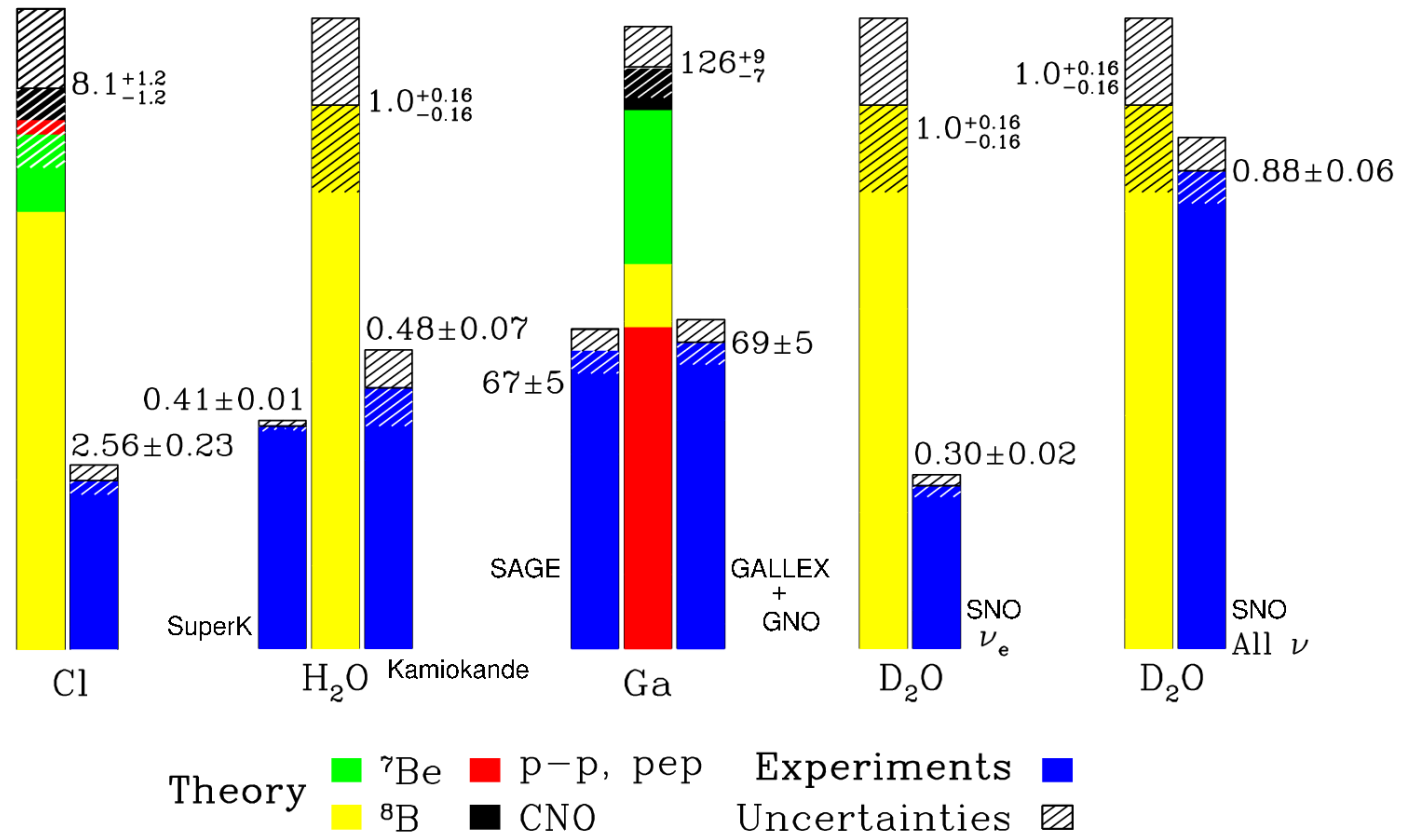


(John Bahcall)

Solar Neutrinos

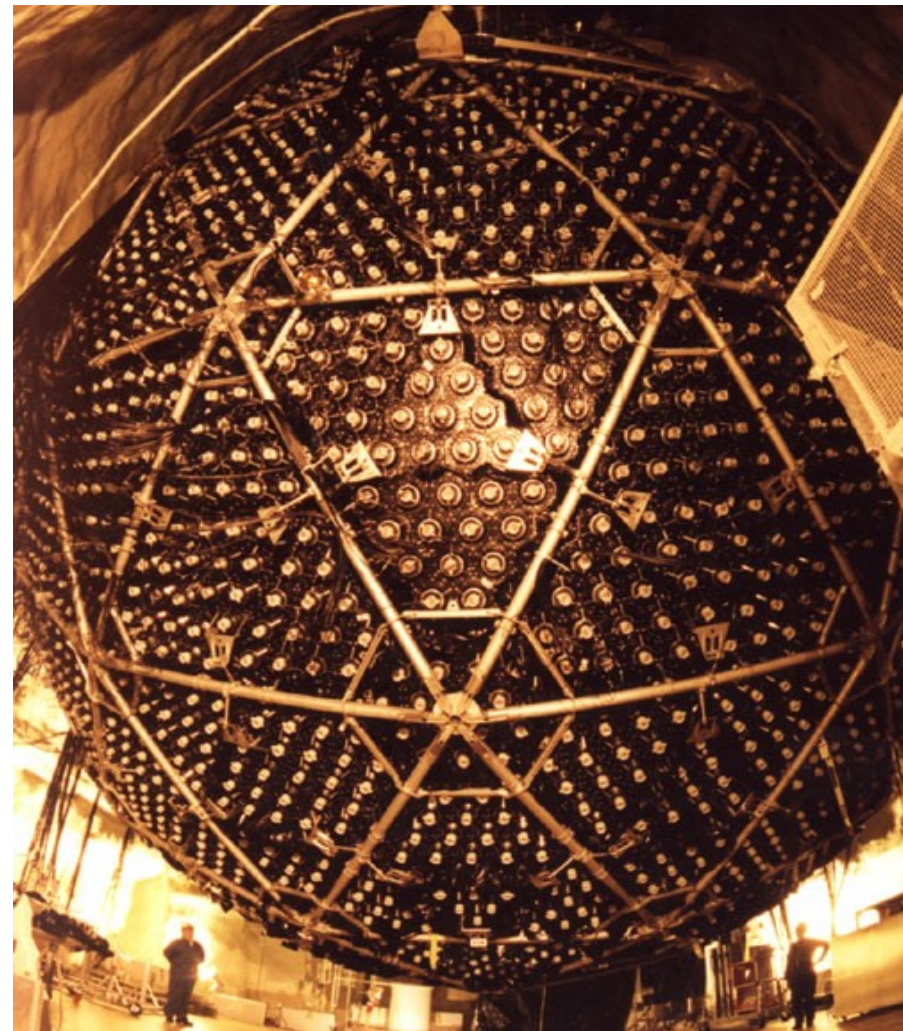
- The solar neutrino problem went on for over 30 years—each experiment was only measuring a small fraction of the expected amount of neutrinos.

Total Rates: Standard Model vs. Experiment
Bahcall–Serenelli 2005 [BS05(OP)]



Solar Neutrinos

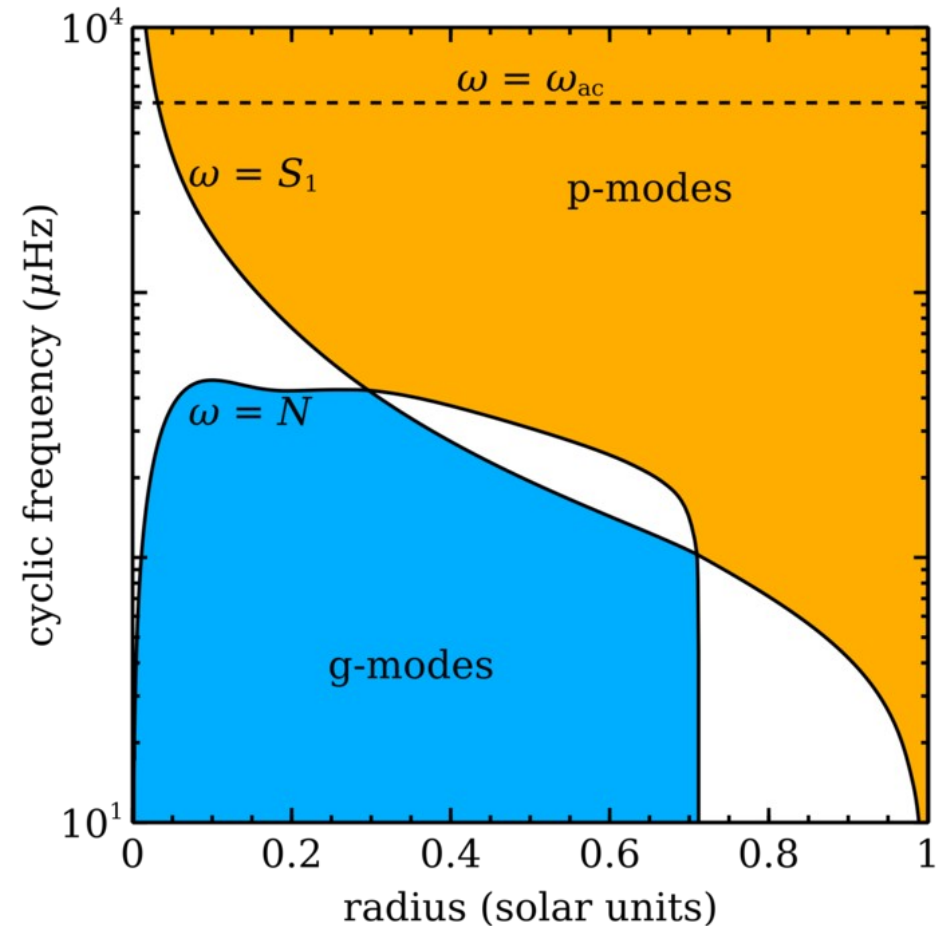
- Sudbury Neutrino Observatory uses 1,000 tons of heavy water in a 12 m sphere surrounded by photomultiplier tubes.
- SNO is also sensitive to all types of neutrinos.
 - Can also distinguish between electron neutrinos and others
- Found some of the electron neutrinos produced in the Sun changed into other types of neutrinos along the way.
- Theoretical physics: neutrinos must have mass.



(PBS)

Helioseismology

- Study internal structure of Sun by observing surface oscillations
 - p modes (sound waves)
 - Frequency between 1 and 5 mHz
 - Think of ringing via spherical harmonics
 - g modes (gravity waves)
 - Only present in stably stratified regions
 - No interior g-modes have been detected in the Sun
 - f modes (surface gravity waves)
- Observing p-modes essentially tells you about the sound speed in the convective zone
 - Inverse problem



Helioseismology

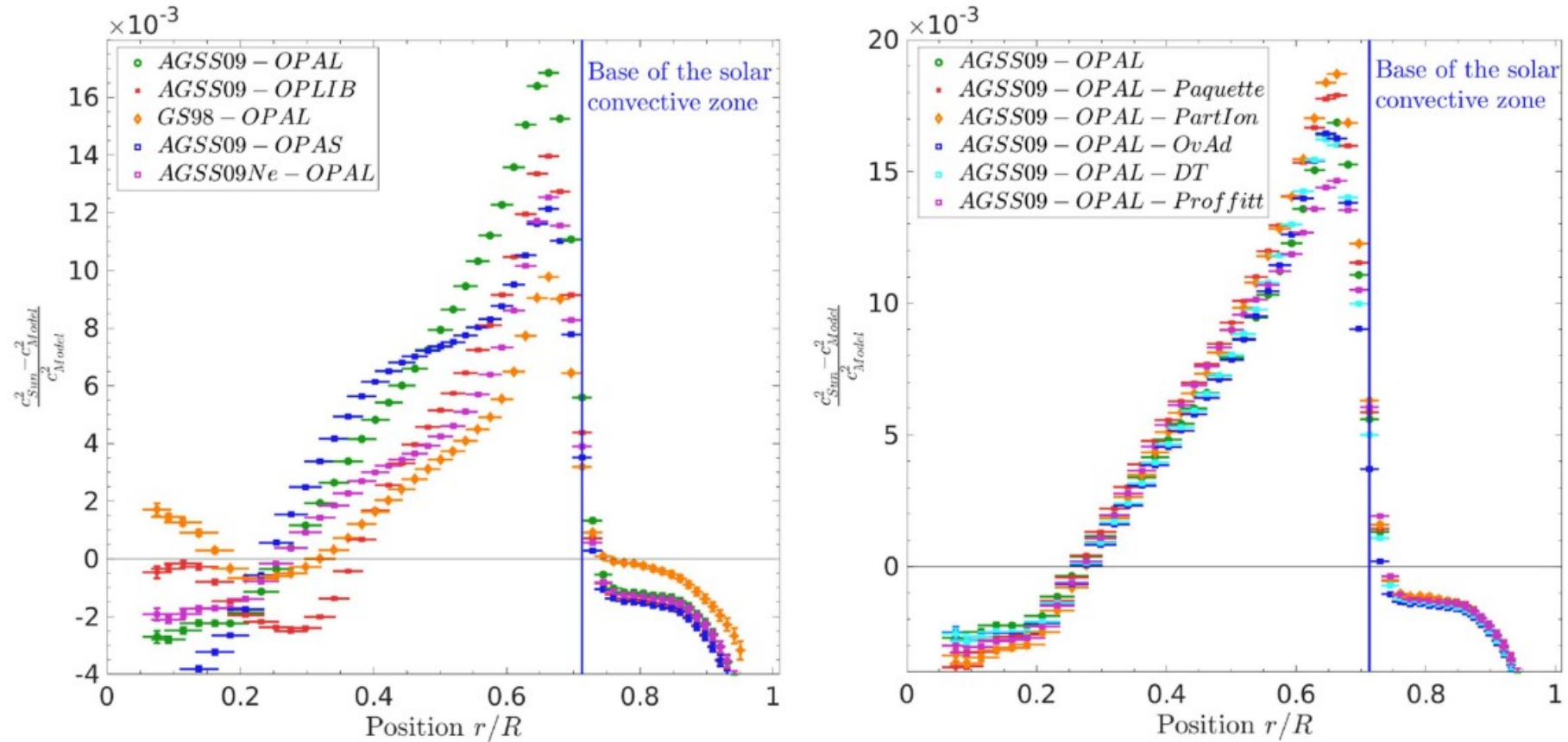
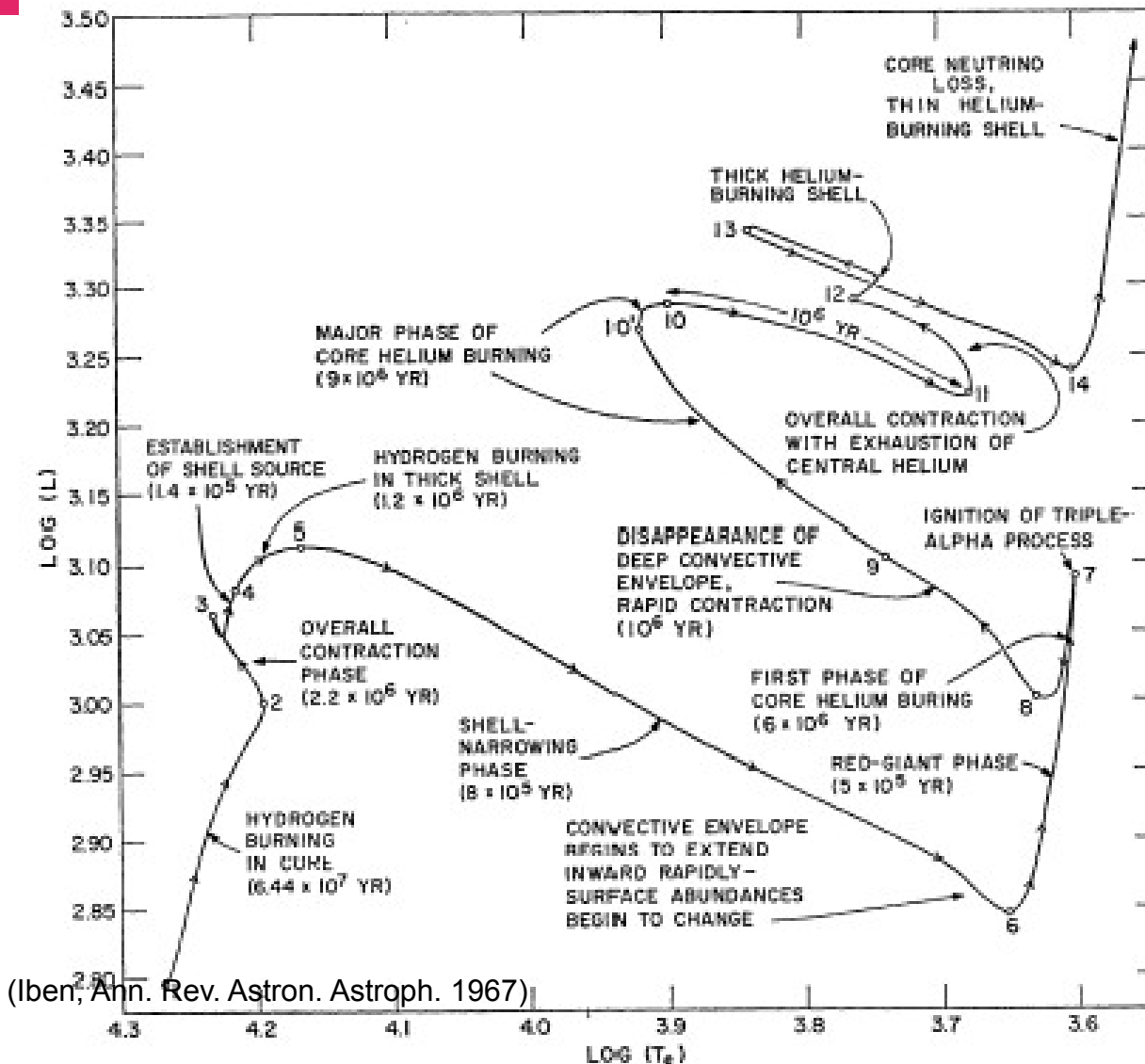


FIGURE 2 | (Left) Relative squared sound speed differences between standard solar models using various abundance and opacity tables and helioseismic results. **(Right)** Relative squared sound speed differences between models including various prescriptions for the mixing of the chemical elements and helioseismic results.

Helioseismology

- Broad agreement with solar models
- Differences blamed on:
 - Opacities
 - Mixing of chemical elements

Sun's Evolution

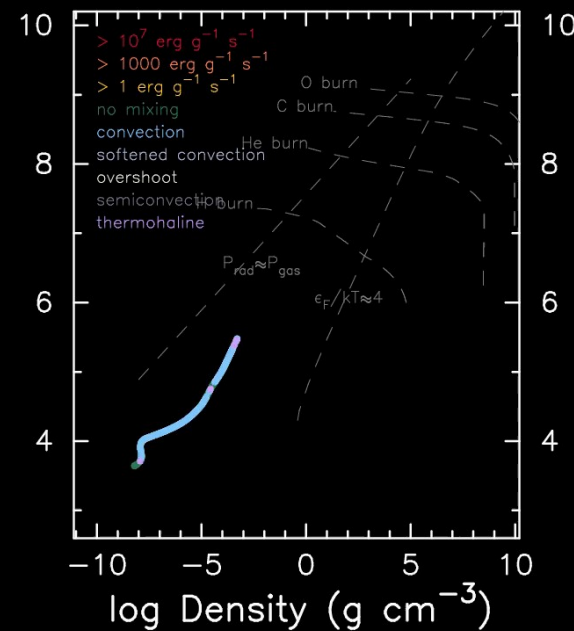
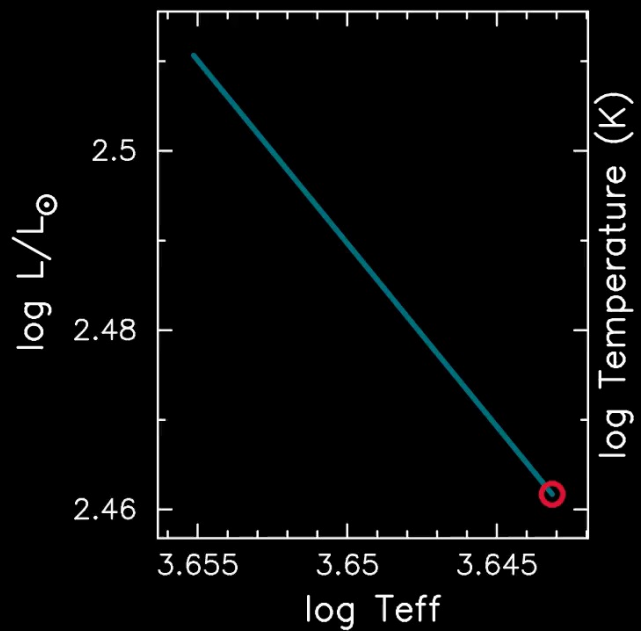
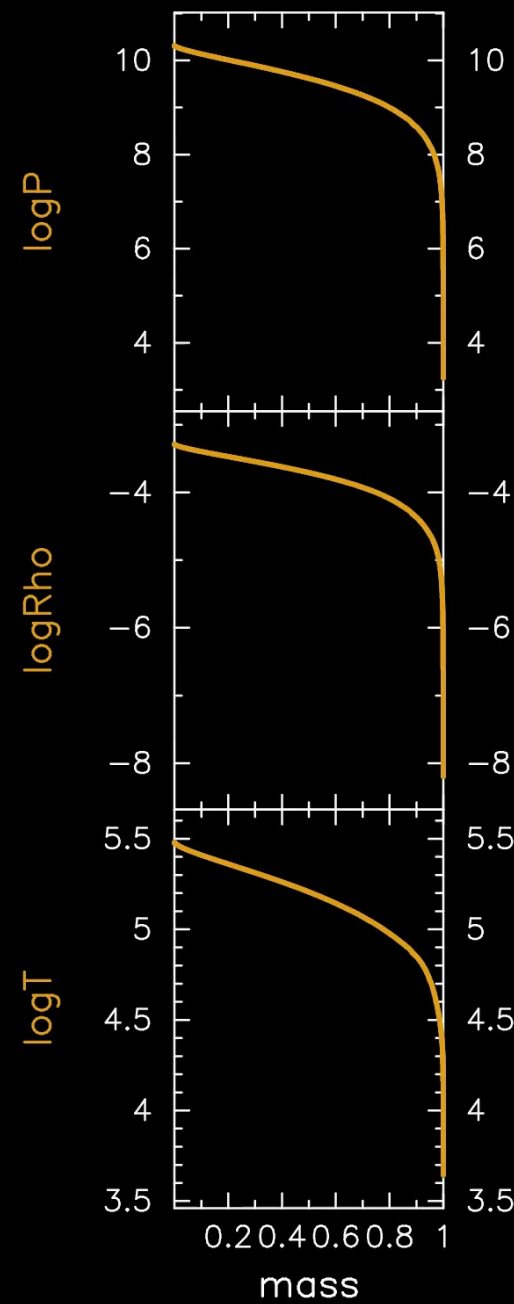
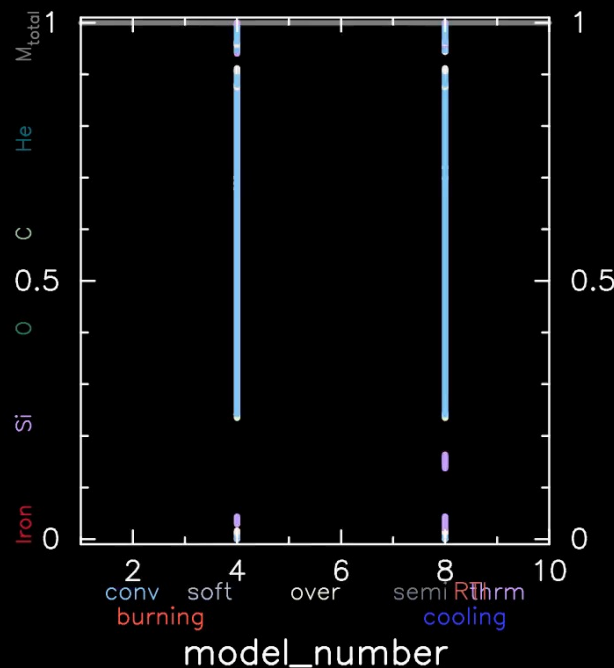
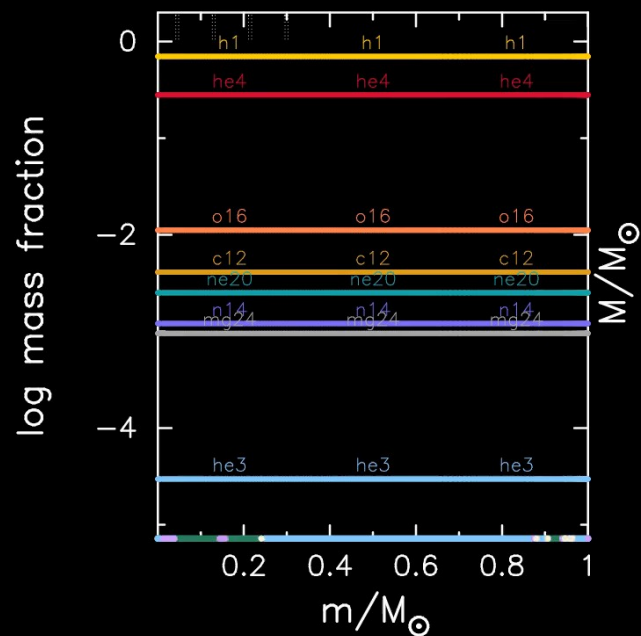


(Iben, Ann. Rev. Astron. Astroph. 1967)

age $2.595868e-4$ yrs

model 10

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$1 M_{\odot}$