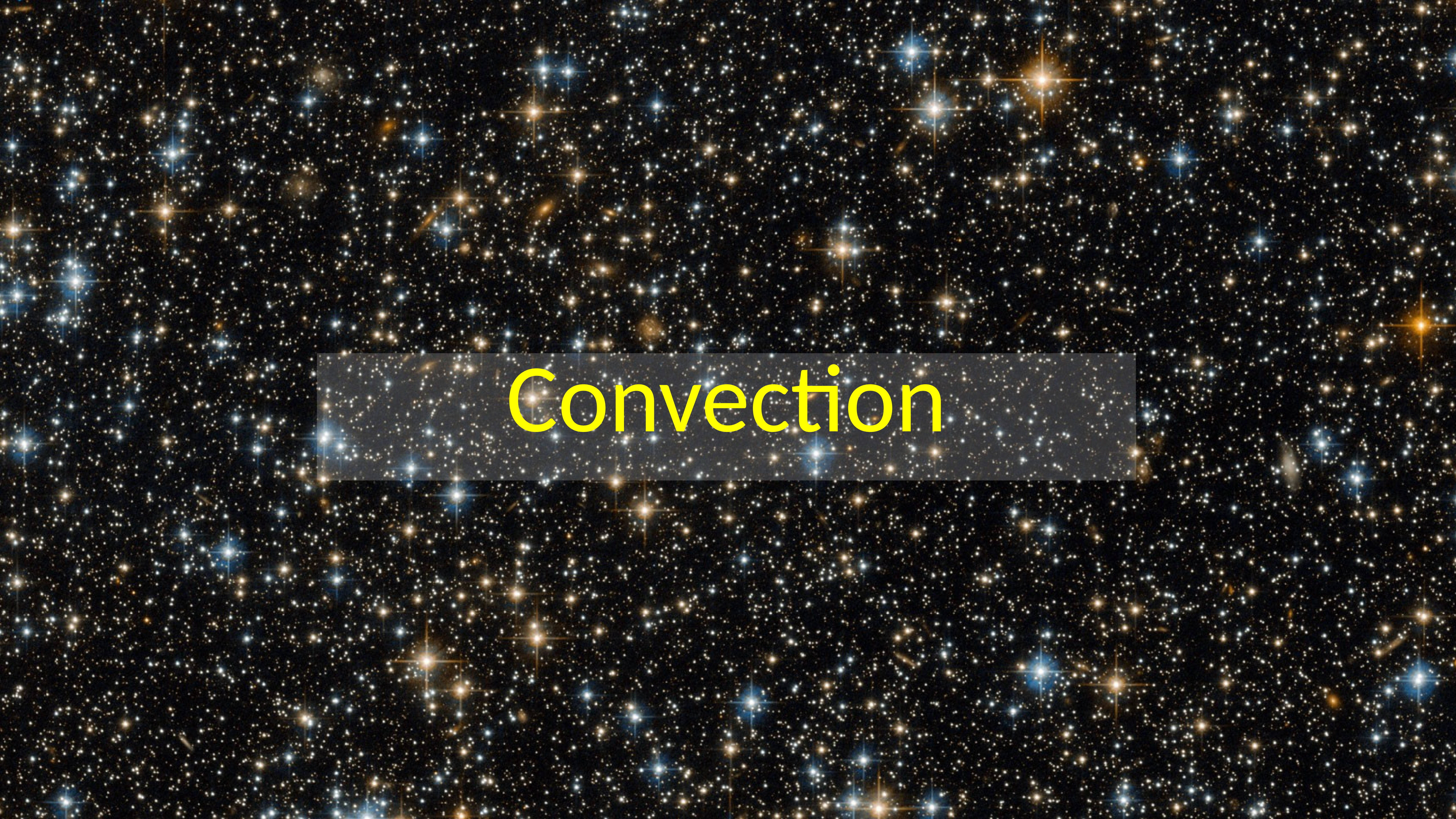
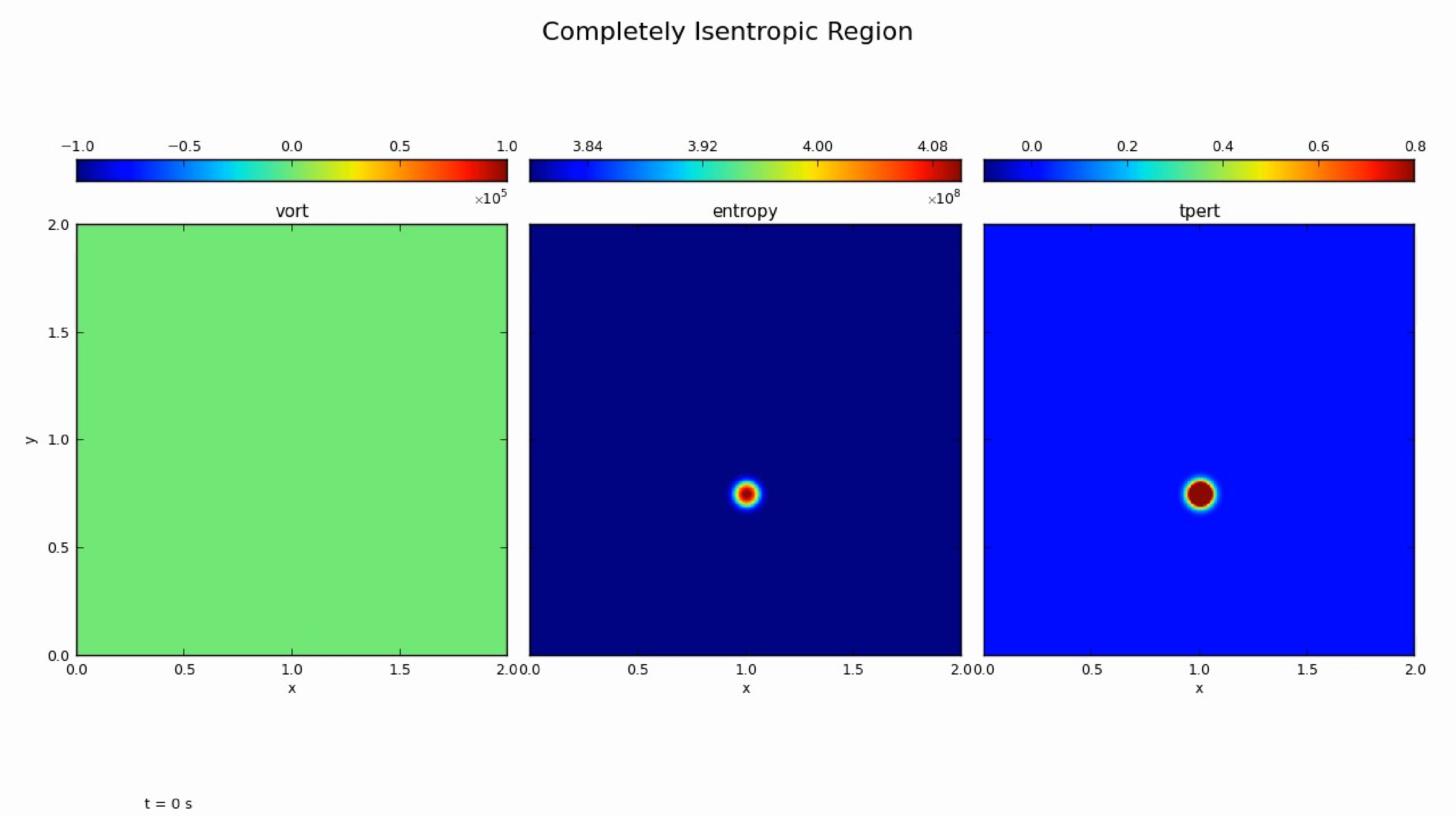


Convection

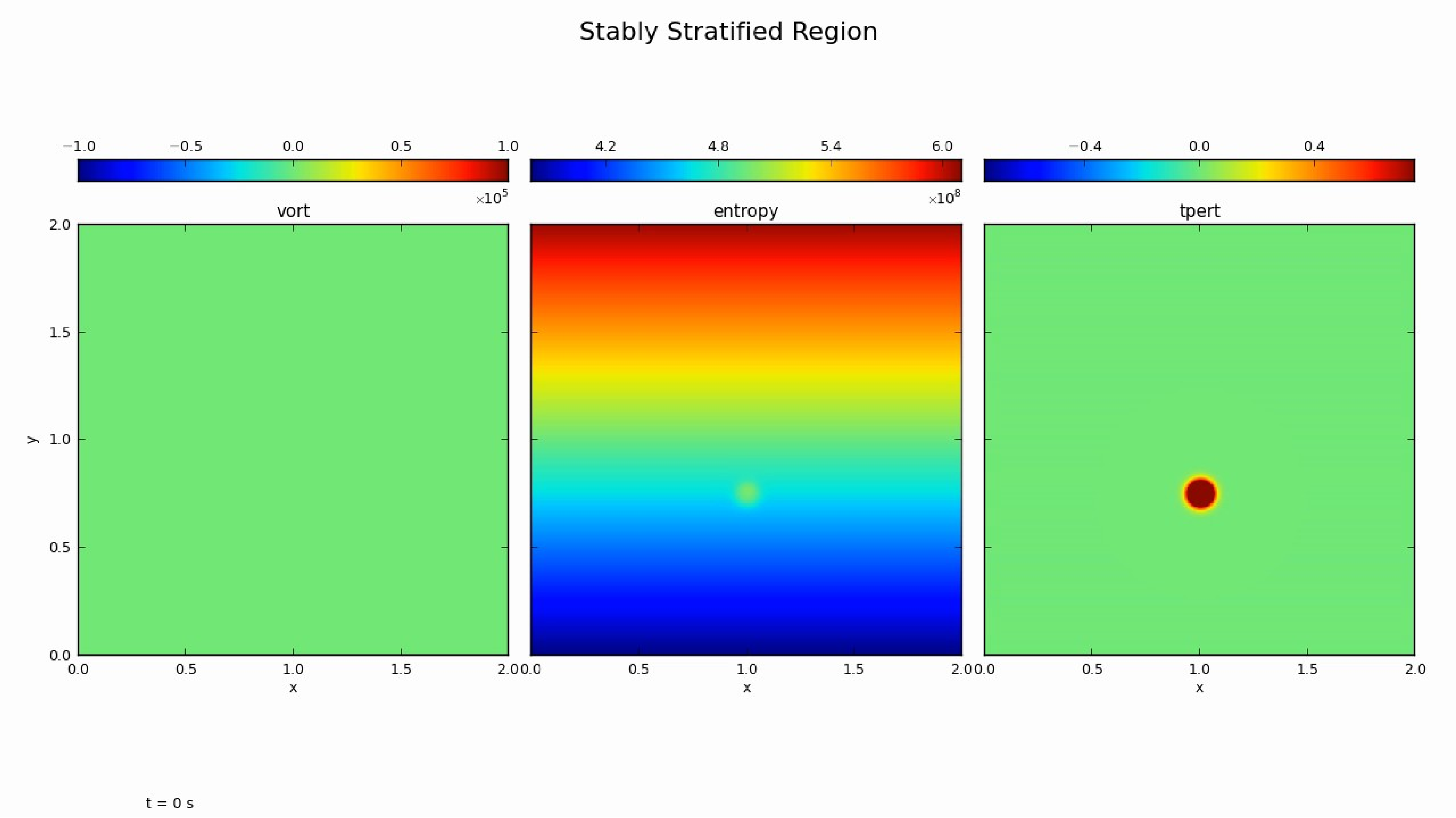
Convection

- We are straying a bit from HKT Ch 5, but it is still a good read
- Carroll & Ostlie Ch. 10 also serve as a reference for these notes

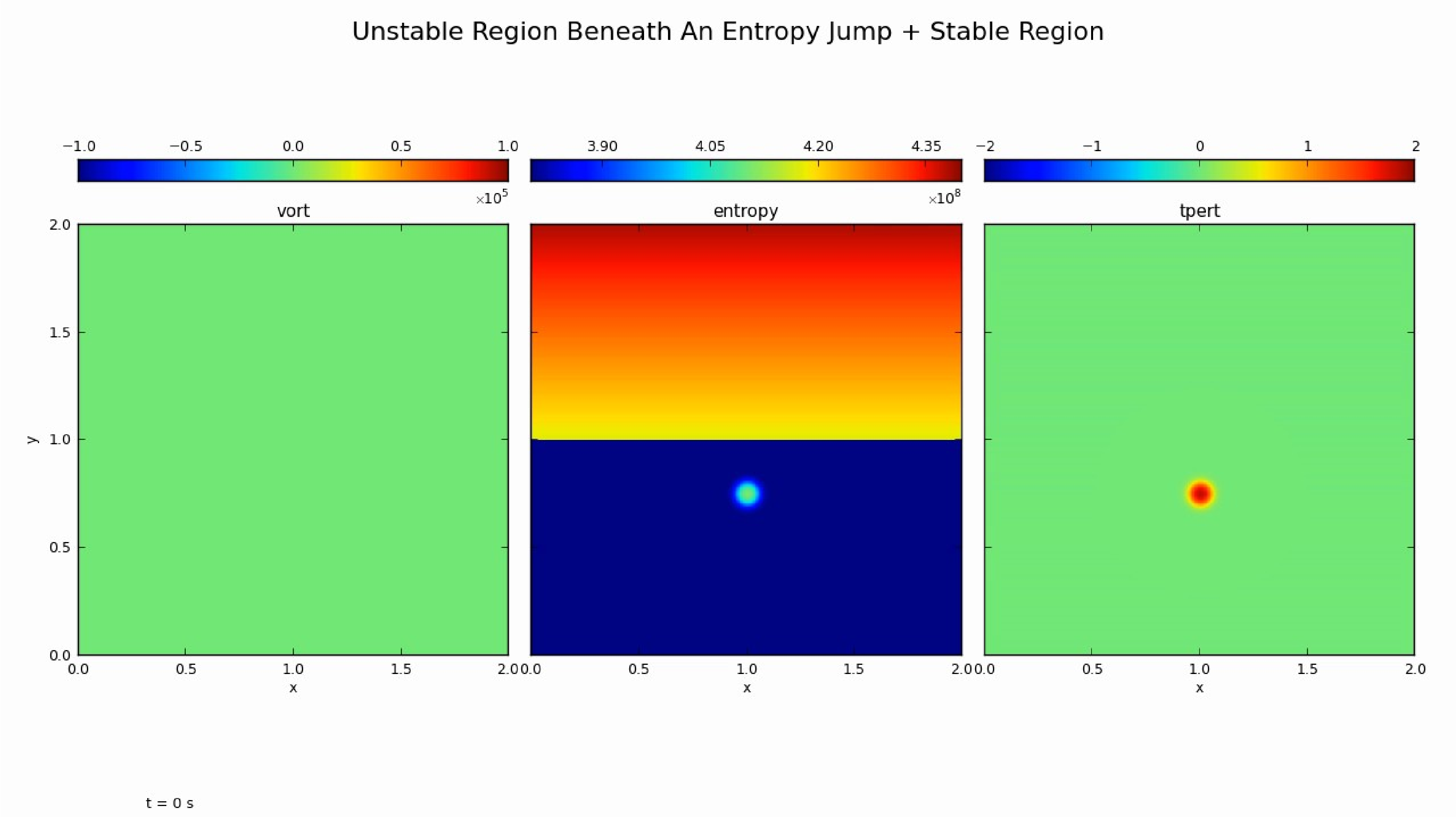
The Role of Entropy



The Role of Entropy



The Role of Entropy



Semi-Convection

- What about composition gradients?

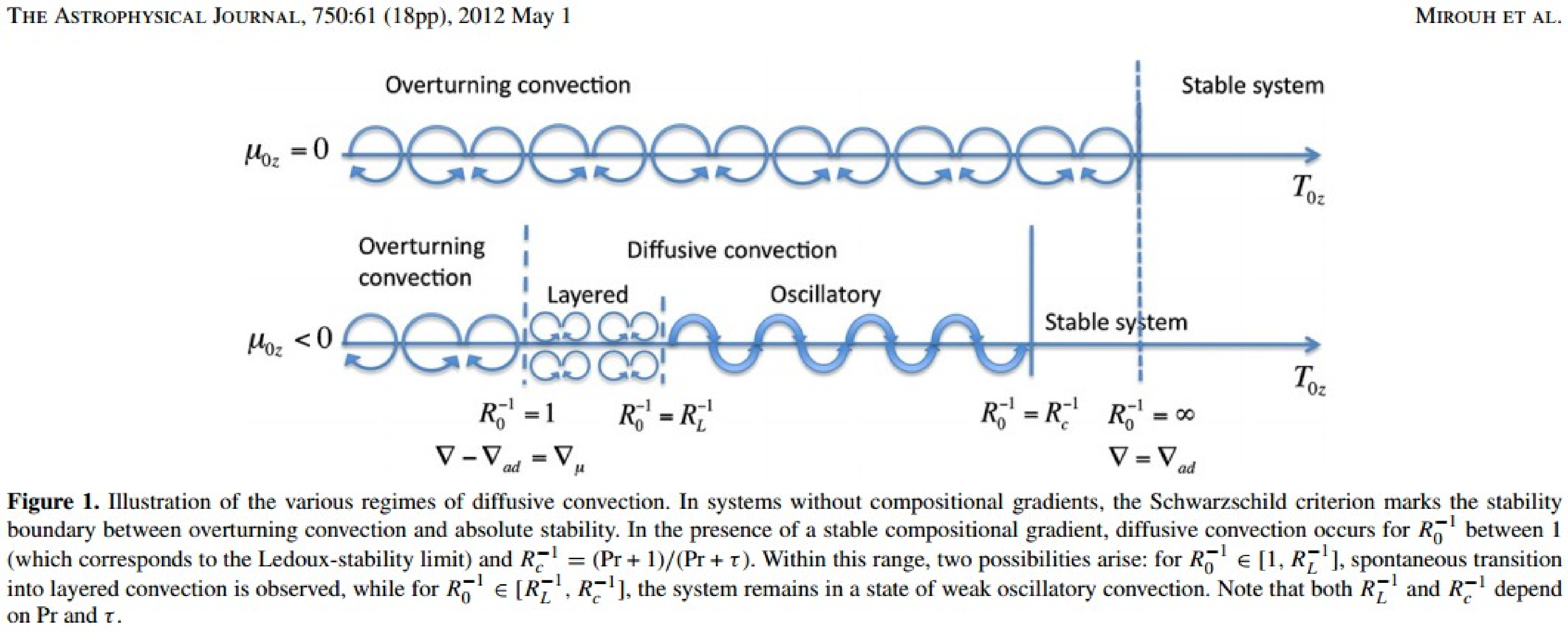


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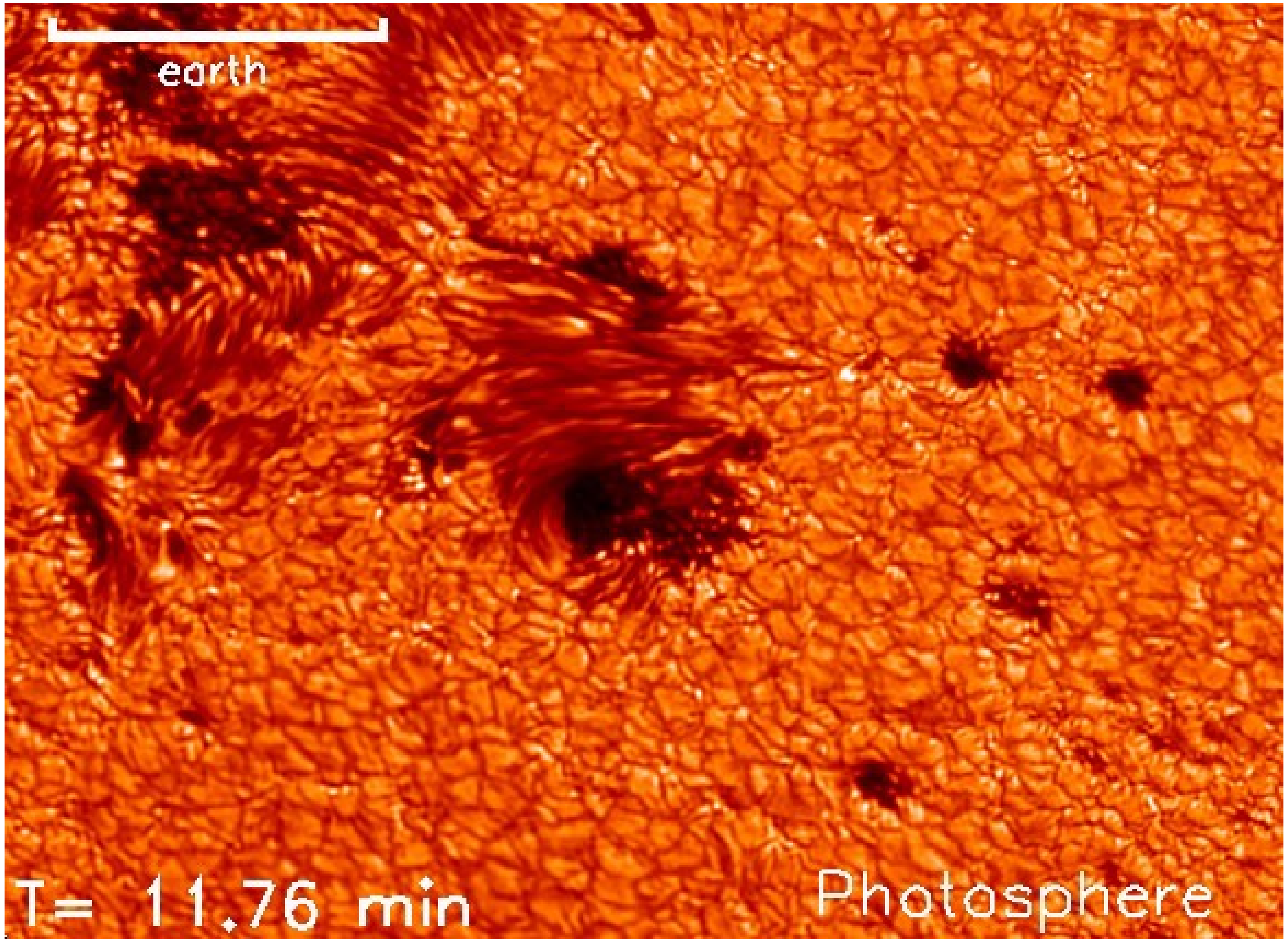
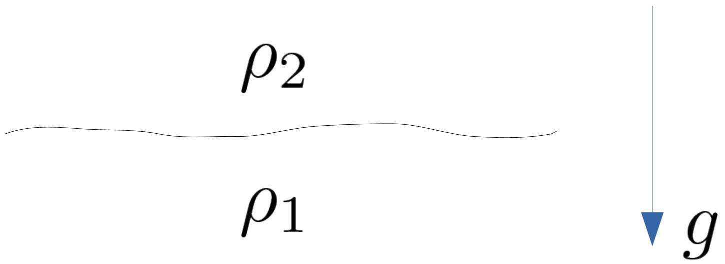
Figure 1. Illustration of the various regimes of diffusive convection. In systems without compositional gradients, the Schwarzschild criterion marks the stability boundary between overturning convection and absolute stability. In the presence of a stable compositional gradient, diffusive convection occurs for R_0^{-1} between 1 (which corresponds to the Ledoux-stability limit) and $R_c^{-1} = (\mathrm{Pr} + 1)/(\mathrm{Pr} + \tau)$. Within this range, two possibilities arise: for $R_0^{-1} \in [1, R_L^{-1}]$, spontaneous transition into layered convection is observed, while for $R_0^{-1} \in [R_L^{-1}, R_c^{-1}]$, the system remains in a state of weak oscillatory convection. Note that both R_L^{-1} and R_c^{-1} depend on Pr and τ.

Granulation—Convection in the Sun

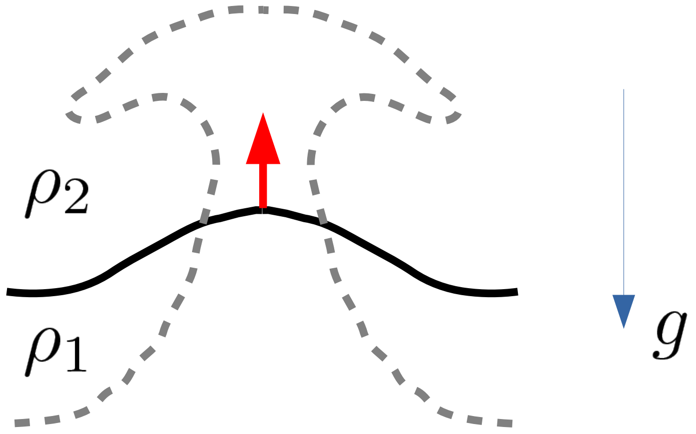


Rayleigh-Taylor Instability

- Light fluid beneath a dense fluid in gravitational field is unstable
  - Bubbles of light fluid buoyantly rise
  - Spikes of dense fluid fall
  - Mixes!



$$\rho_2 > \rho_1$$

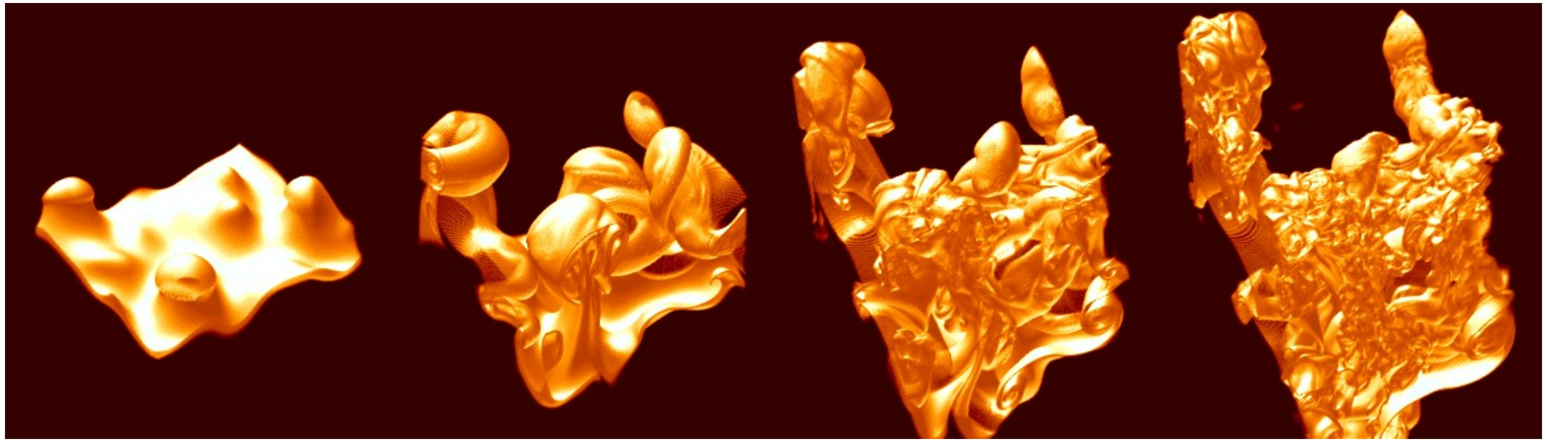


Rayleigh-Taylor Instability

- (Linear) growth rate:

$$\omega^2 = kg\frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$$

  - Smallest scales grow fastest



Rayleigh-Taylor Instability

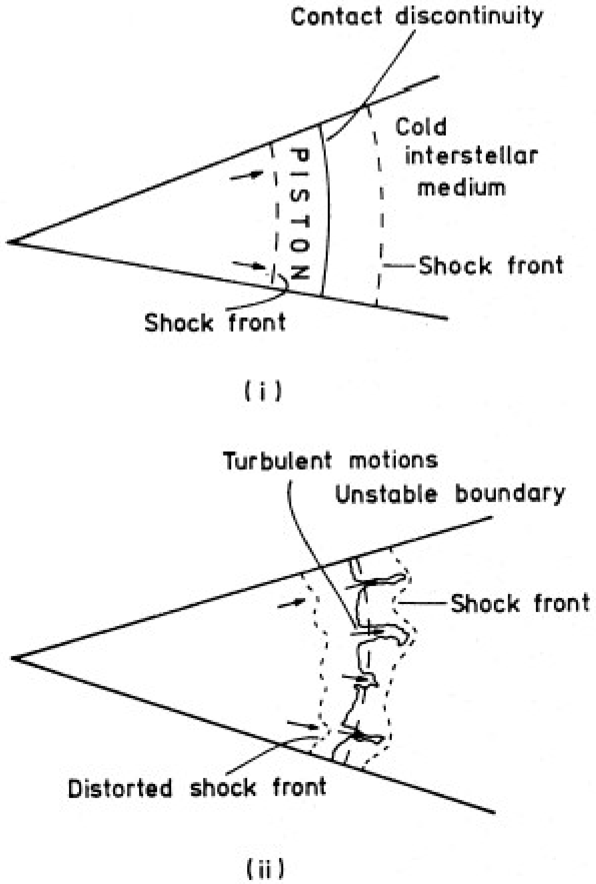
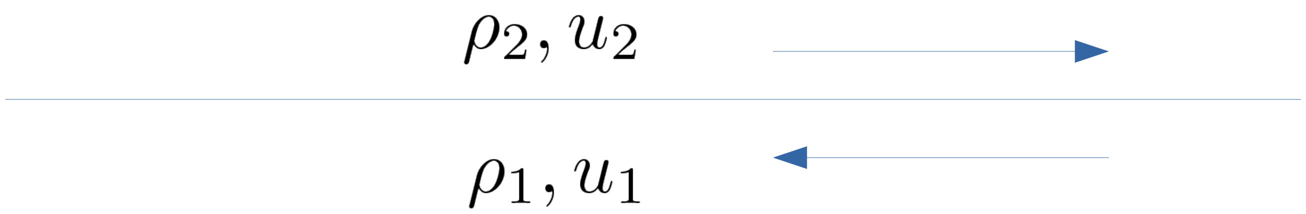


FIG. 3. (1) *Schematic structure of a young supernova remnant, showing the internal shock front.* (2) *Modification of internal structure when the contact discontinuity is distorted by the Rayleigh–Taylor instability. Some fraction of the energy now appears as random motions in the neighbourhood of the filaments.*

(from Gull 1975)

Kelvin-Helmholtz Instability

- Two fluids moving past one another
  - Shear at the interface is unstable to perturbations



ρ_2, u_2

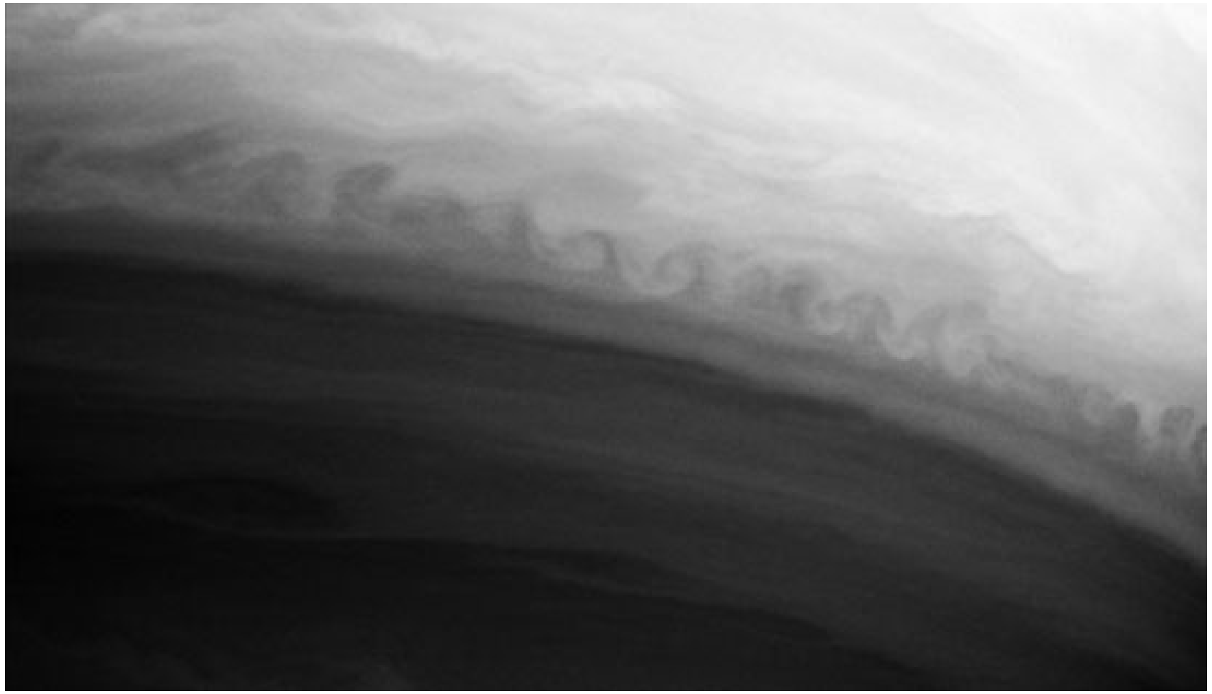
ρ_1, u_1

  - Instability no matter how small the velocity difference
  - Linear growth rate:

$$\omega = k \frac{\sqrt{\rho_1 \rho_2}}{(\rho_1 + \rho_2)} \Delta u$$

  - Helmholtz:
    - *Every perfectly geometrically sharp edge by which a fluid flows must tear it asunder and establish a surface of separation, however slowly the rest of the fluid may move*

Kelvin-Helmholtz Instability



Turbulent boundary between two latitudinal bands in Saturn's atmosphere shows K-H features.

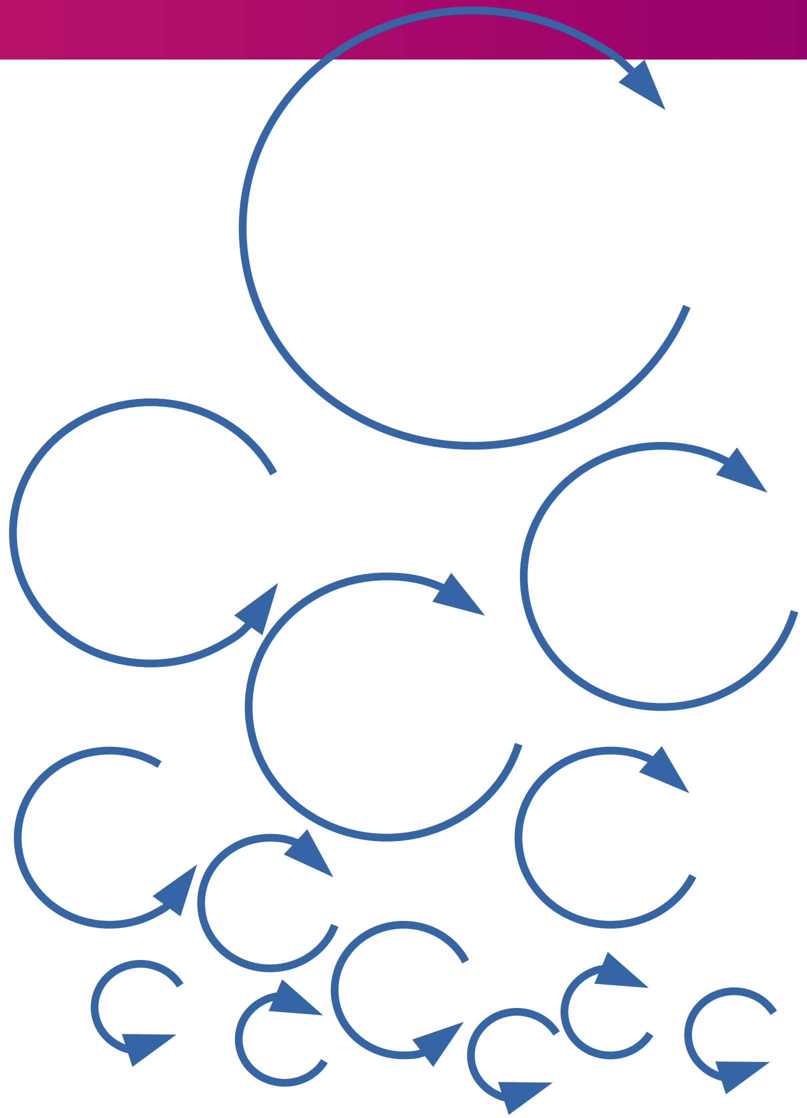
(NASA/Cassini)

K-H is also active in cloud formation on earth.

(Patrick Witting)

Turbulence

- In a turbulent flow, energy is input on large scales, driving vortical motions, which cascade down to smaller and smaller scales…
- Turbulent flows exhibit:
  - Random motions—velocity at any point cannot be predicted, but statistical properties can be determined
  - Scale-free behavior on scales larger than the viscous dissipation scale
  - Dissipation will decrease the kinetic energy with time—driving needed to sustain the turbulence



Turbulence

(Shu, Ch. 9)

- Rate at which energy is fed into the largest scales is:

$$\epsilon = \frac{U^2}{L/U} = \frac{U^3}{L}$$

  - In steady state, this energy is transferred to eddies of smaller and smaller scales (through the U·∇U term).
- On scale λ, energy that cascades through eddies is

$$\epsilon \sim \frac{u_\lambda^3}{\lambda}$$

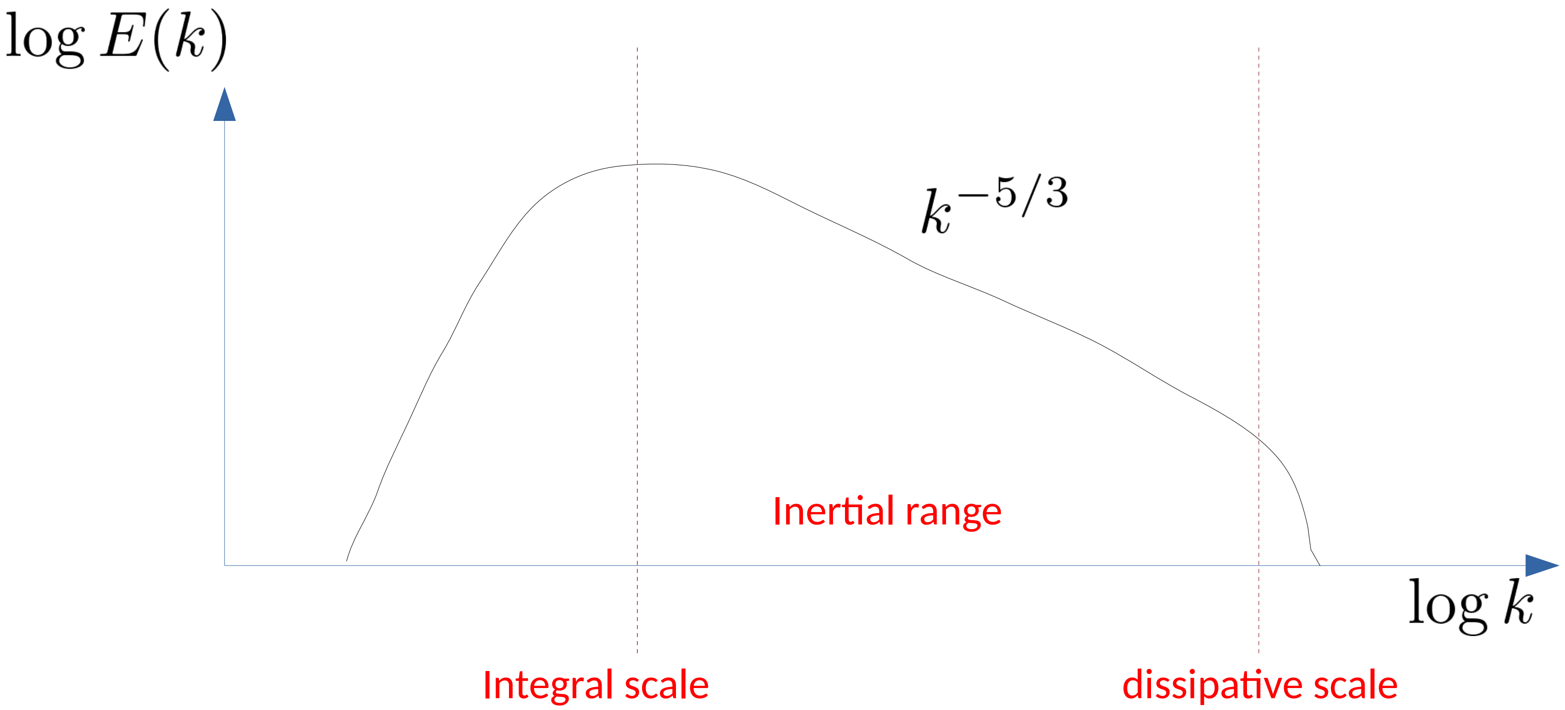
- Kolmogorov: kinetic energy flux is constant through all scales

$$u_\lambda = U \left(\frac{\lambda}{L} \right)^{1/3}$$

- This is Kolmogorov's law.
  - Largest eddies have most of the velocity (turbulent energy)
  - Smallest eddies carry most of the vorticity.

Energy Cascade

(Shu, Ch. 9)



Turbulence plays an important role in convection in stars, in stellar and galactic magnetic dynamos, and the dynamics of the interstellar medium.