Close Binary Evolution, Novae, and XRBs

General Binary System Orbits

- Obey a generalized form of Kepler's laws of planetary motion
  - Orbits are ellipses
  - Sweep out equal areas in equal time (conservation of angular momentum)
  - Harmonic relation between the period and semi-major axis:

$$\frac{4\pi^2 a^3}{G} = (M_1 + M_2)P^2$$

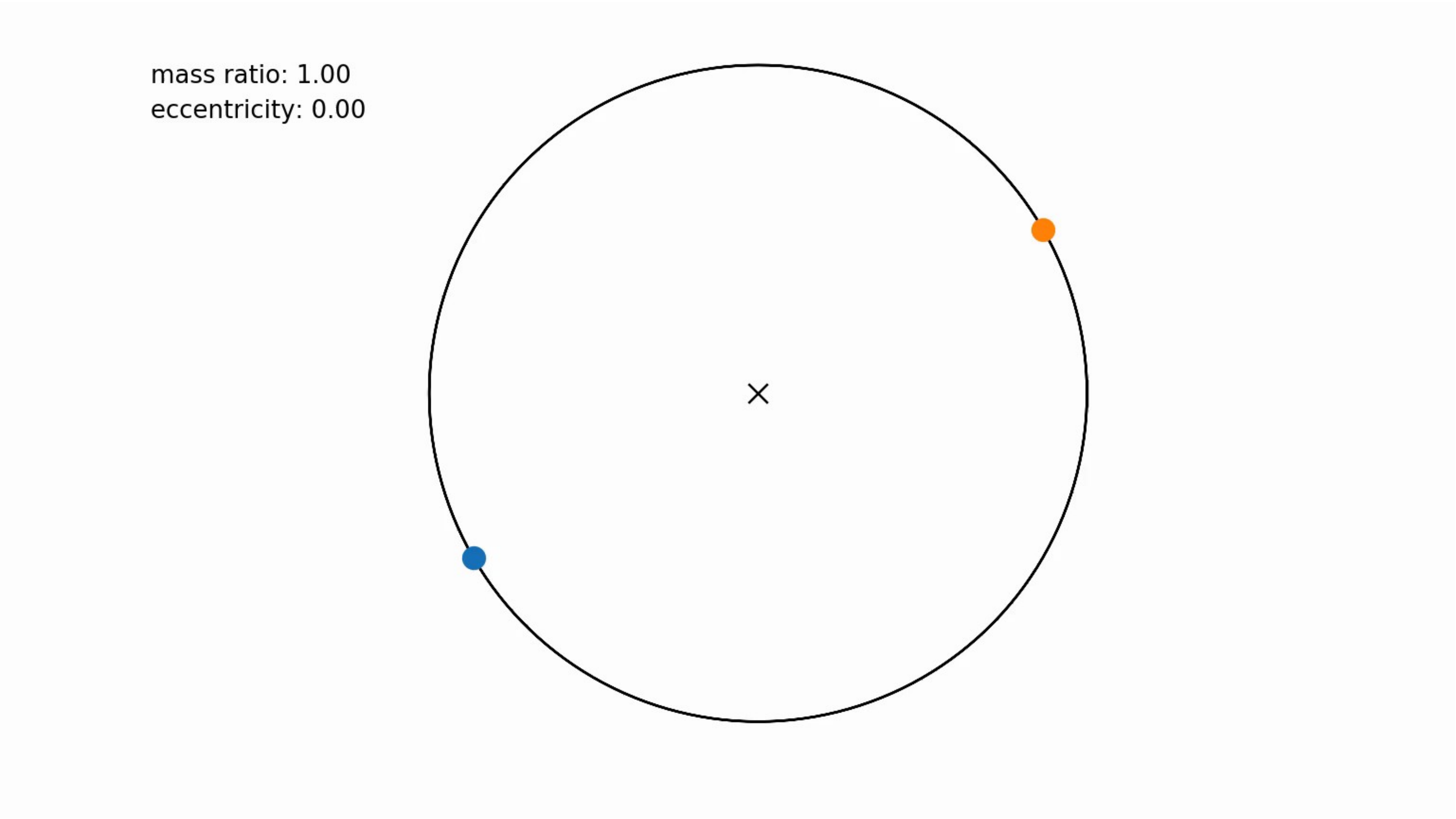
- Center of mass condition:

$$M_1 a_1 = M_2 a_2$$

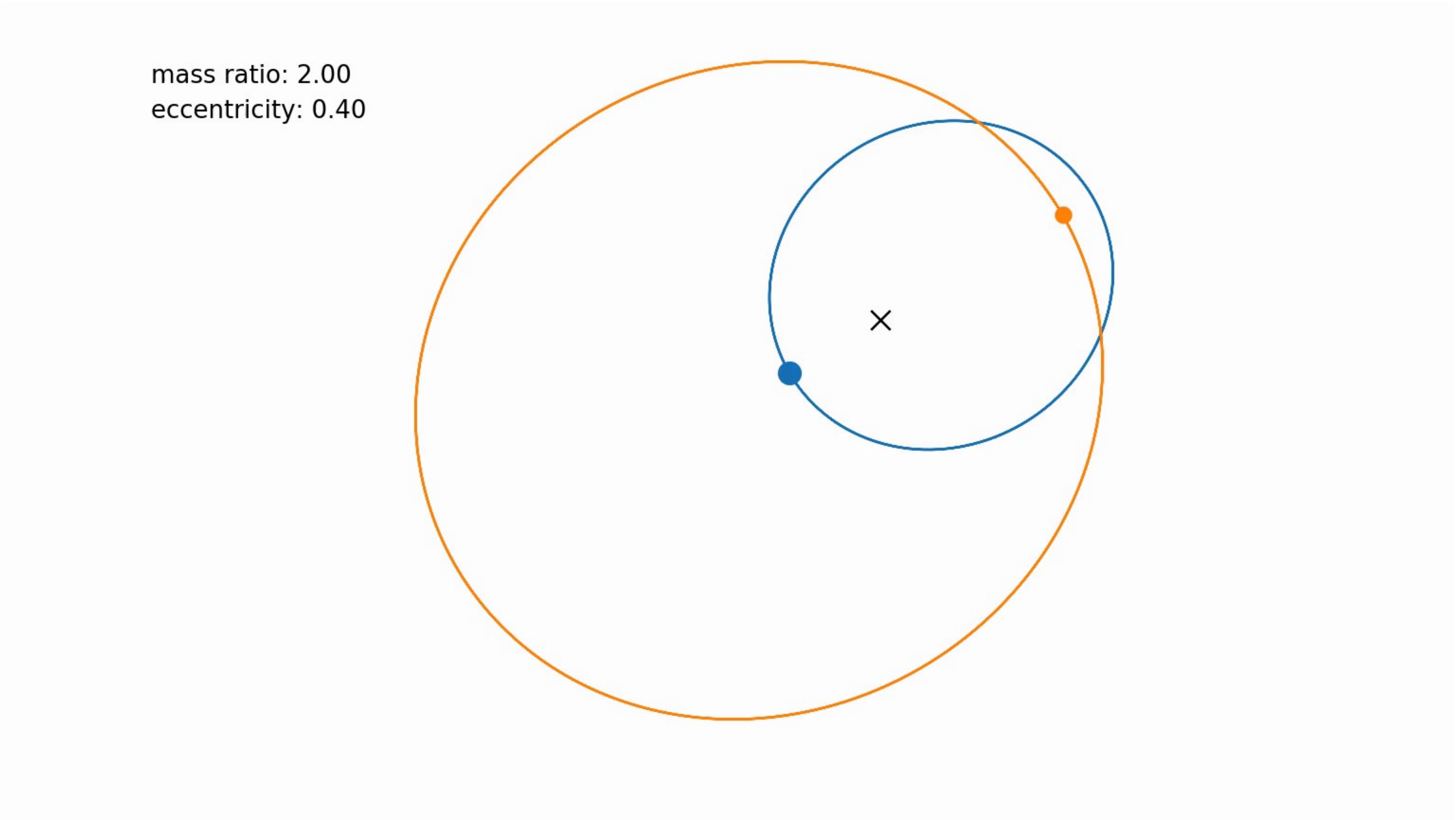
$$a = a_1 + a_2$$

- Eccentricity of both orbits needs to be the same
  - In frame of one star, the eccentricity of the relative position of the other star also has the same eccentricity

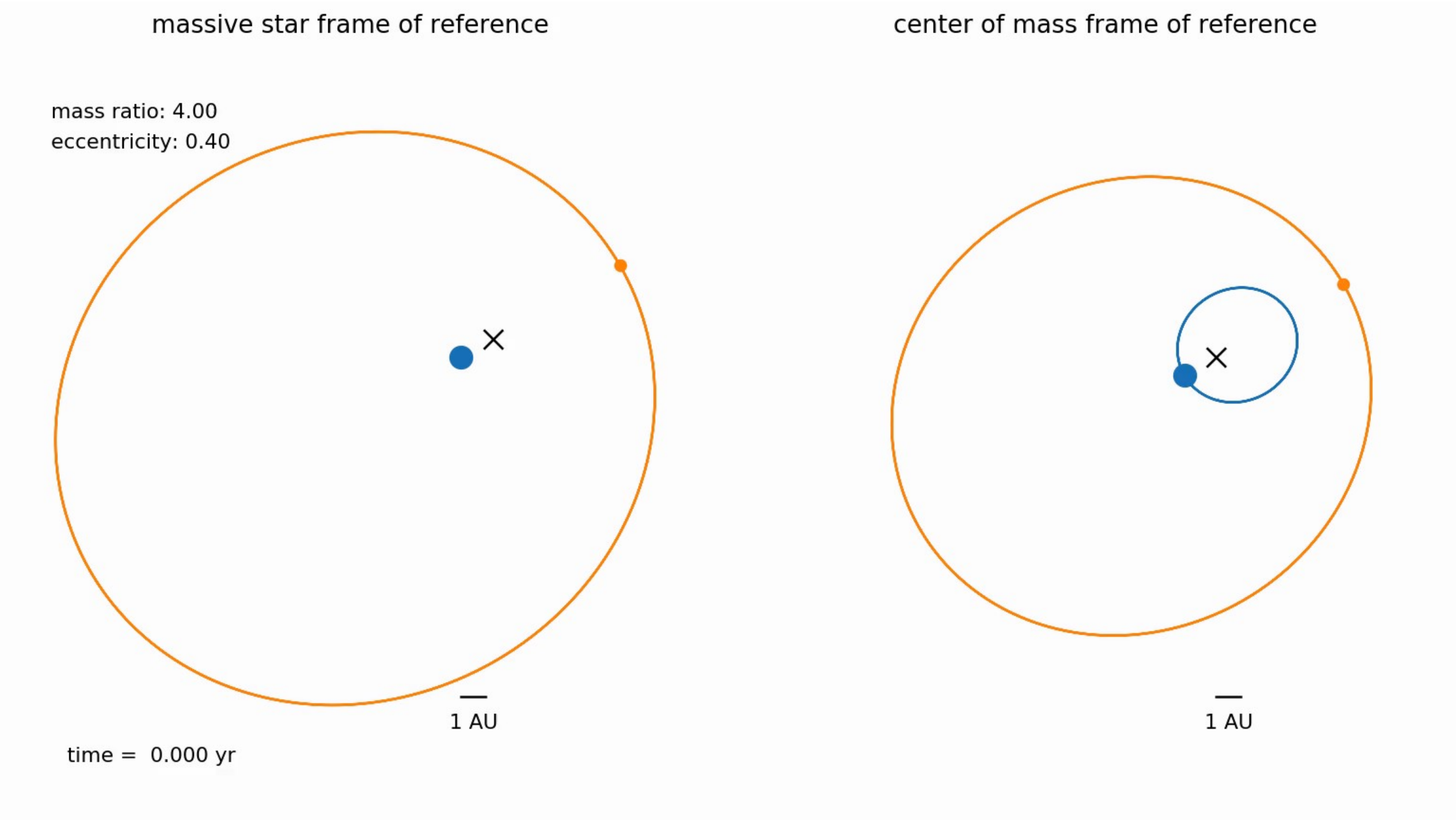
Equal Mass



Unequal Mass



Frame of Reference

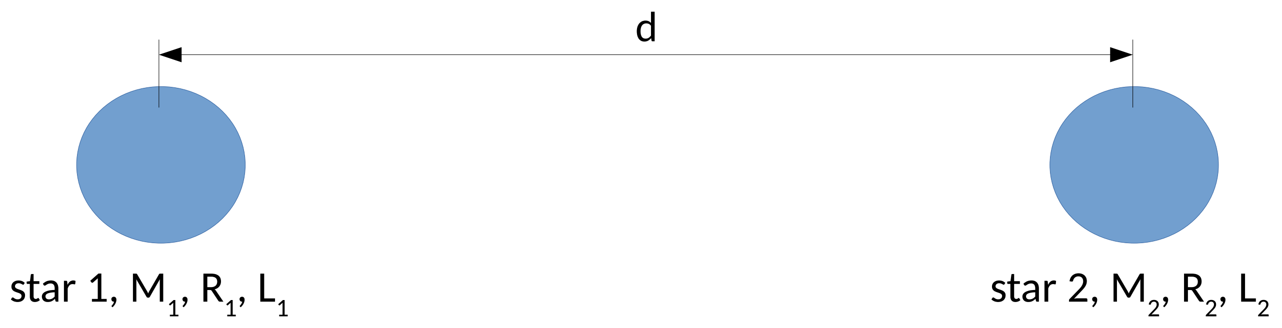


What Are the Consequences?

- Hotter star can irradiate its companion → outer layers expand
- Tides can distort the stars
- Mass transfer

Irradiation

- Consider two stars separated by distance d



  - Star 1 absorbs energy from star 2's irradiation:

$$P_{\mathrm{abs}} = \frac{L_2}{4\pi d^2} \cdot \pi R_1^2$$

  - In equilibrium, star 1 re-radiates this, along with the L_1 from its interior

$$L = L_1 + L_2 \frac{\pi R_1^2}{4\pi d^2}$$

Irradiation

- Effective temperature is:

$$T_{\rm eff} = T_{\rm eff,1}\left[1 + \left(\frac{T_{\rm eff,2}}{T_{\rm eff,1}}\right)^4 \left(\frac{R_2}{2d}\right)^2\right]^{1/4}$$

  - Here, $T_{\rm eff,1}$ is the effective temperature in the absence of a companion
  - This correction is usually quite small
- What is the scale in the star that feels this temperature change?

$$\chi = \frac{T}{|dT/dm|} = \frac{4acT^4(4\pi r^2)^2}{3\kappa F}$$

  - Here, χ is a mass scale

Irradiation

- Taking F ~ L, T ~ T_{eff}, r ~ R_1, and using a = 4σ/c, we have

$$\chi = \frac{16}{3}\frac{4\pi R_1^2}{\kappa}$$

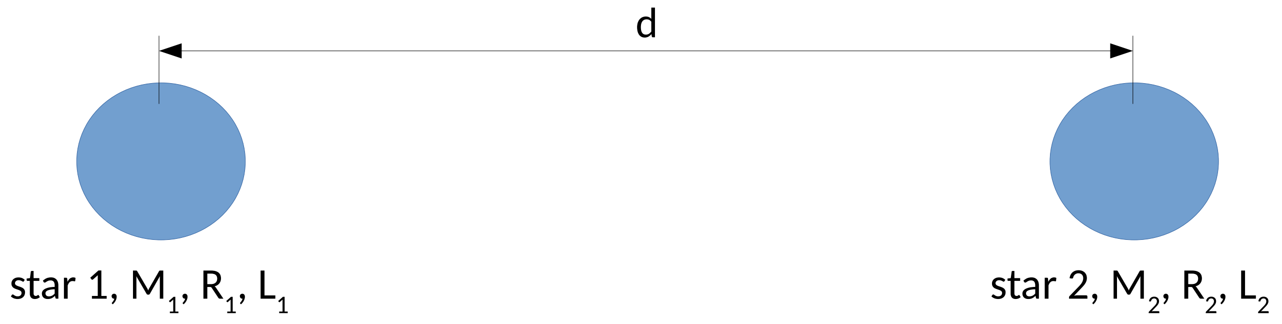
- Now, normalize by the total mass, we have:

$$\frac{\chi}{M_1} = \frac{16}{\kappa\rho R_1}\frac{\rho}{\bar{\rho}} \sim 16\frac{\rho}{\bar{\rho}} \ll 1$$

  – Where we recognize $\kappa\rho R_1$ is the optical depth using photosphere conditions and is > 1
- Most of the mass of the star is unaffected by the irradiation
  – We expect the core, energy generation, and stellar evolution to not change significantly

Tidal Interactions

- How much does a star affect its companion gravitationally?



  - Force / unit mass (acceleration) exerted on star 1 by star 2
    - At the center: GM_2/d^2
    - Some distance r away from the center (along line connecting stars):

$$\frac{GM_2}{(d-r)^2}$$

    - Difference is stretching (assuming $r/d \ll 1$):

$$f_{\rm tide}(r) = \frac{GM_2}{(d-r)^2} - \frac{GM_2}{d^2} \sim \frac{2GM_2 r}{d^3}$$

Tidal Interactions

- What is the "tidal pressure" corresponding to this?

$$P_{\mathrm{tidal}} \sim \int f_{\mathrm{tide}}(r)\rho(r)dr \sim \frac{3GM_1M_2}{4\pi R_1 d^3}$$

- Hydrostatic pressure (from stellar structure equations)

$$P_{\mathrm{hse}} \sim \frac{GM_1\chi}{4\pi R_1^4}$$

  - At the base of the surface layer of mass χ
- Affected region found via $P_{\mathrm{tidal}} = P_{\mathrm{hse}}$:

$$\frac{\chi}{M_1} \sim \frac{M_2}{M_1}\left(\frac{R_1}{d}\right)^3$$

  - Insignificant for large separations

Mass Transfer

- Effects become more pronounced for closer stars
- If the stars are really close, then mass transfer can happen between them.
- Consider spherically symmetric accretion onto star of mass M and radius R, accrete a mass δm
  - Starting infinitely far away, gravitational potential energy release is:

$$\delta E_{\rm grav} = \frac{GM\delta m}{R}$$

  - If we accrete over time δt, accretion rate is $\dot{M} = \delta m/\delta t$

$$\dot{E}_{\rm grav} = \frac{GM\dot{M}}{R}$$

Mass Transfer

- Thermal equilibrium means we radiate this energy away

$$L_{\rm acc} = \dot{E}_{\rm grav} = \frac{GM\dot{M}}{R}$$

  - Ex: Sun doubling in mass over 10^{10} yr needs $\dot{M} = 10^{-10}\ M_\odot$/yr
    - $L_{acc} \sim 10^{-3}\ L_\odot$
    - For a WD or NS, since R is much smaller, the luminosity is much larger

Mass Transfer

- Accretion rate is limited by the Eddington limit
  - This is a 1-d argument, and multi-d effects may allow for super-Eddington accretion
- Eddington luminosity:

$$L_{\rm Edd} = \frac{4\pi c G M}{\kappa} \sim 3.2 \times 10^4 \left(\frac{M}{M_\odot}\right) \left(\frac{\kappa_{\rm es}}{\kappa}\right) L_\odot$$

- We need:

$$L_{\rm acc} = \frac{G M \dot{M}}{R} < \frac{4\pi c G M}{\kappa}$$

$$\dot{M} < \frac{4\pi c R}{\kappa} \sim 10^{-3} \left(\frac{R}{R_\odot}\right) M_\odot \;{\rm yr}^{-1}$$

Mass Transfer

- Radius of a compact object doesn't change much
  - Biggest response: surface T—need to match the accretion luminosity
- Thermal equilibrium

$$T_b = \left(\frac{L_{\rm acc}}{4\pi R^2 \sigma}\right)^{1/4} = \left(\frac{GM\dot{M}}{4\pi R^3 \sigma}\right)^{1/4}$$

  - with the critical mass transfer rate:

$$T_b = \left(\frac{GMc}{R^2 \sigma \kappa}\right)^{1/4}$$

- Typical values:
  - WD (R ~ 0.01 $R_\odot$, M ~ 1 $M_\odot$): T ~ 10^6 K
  - NS (R ~ 10 km, M ~ 1.4 $M_\odot$): T ~ 10^8 K
  - Compact objects are X-ray emitters!

Close Binary Systems

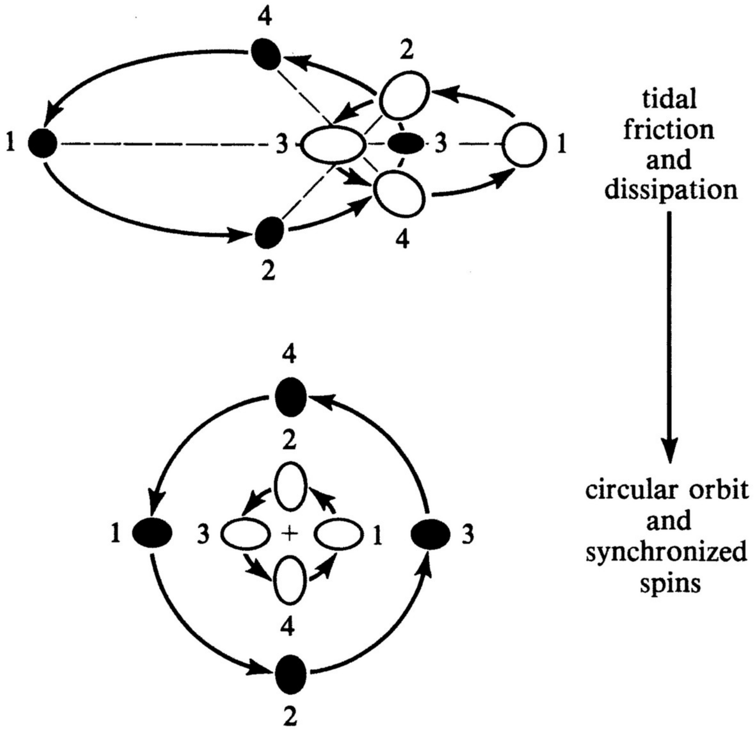


Figure 10.6. When two stars are separated by a distance which is not much greater than their individual sizes, tidal forces can raise bulges in the stars. The tidal interactions lead to a slow change of the orbit shape and the spin rates of the stars. Except in very unusual circumstances, the long-term evolution tends to bring the system to a state of circular orbits and synchronized spins.

(Shu)

- About 1/2 of all stars are in binary systems
- Close binary: one star is able to affect the evolution of the other
  - Tidal effects
  - Mass transfer
- Equilibrium:
  - Tidal distortions dissipate energy
  - Orbits circularize
  - Synchronous rotation

Equipotentials

- Consider two stars in the x-y plane, with CM at the origin
  - We'll assume $M_1 > M_2$
  - Rotating frame: stars stationary
  - Centrifugal force on some test mass: always radial

$$F_c = m\omega^2 r\hat{r}$$

  - We create a "centrifugal potential energy"

$$\Delta U = -\int_{r_i}^{r_f} F_c \cdot dr = -\frac{1}{2}m\omega^2(r_f^2 - r_i^2)$$

  - Zero point at origin:

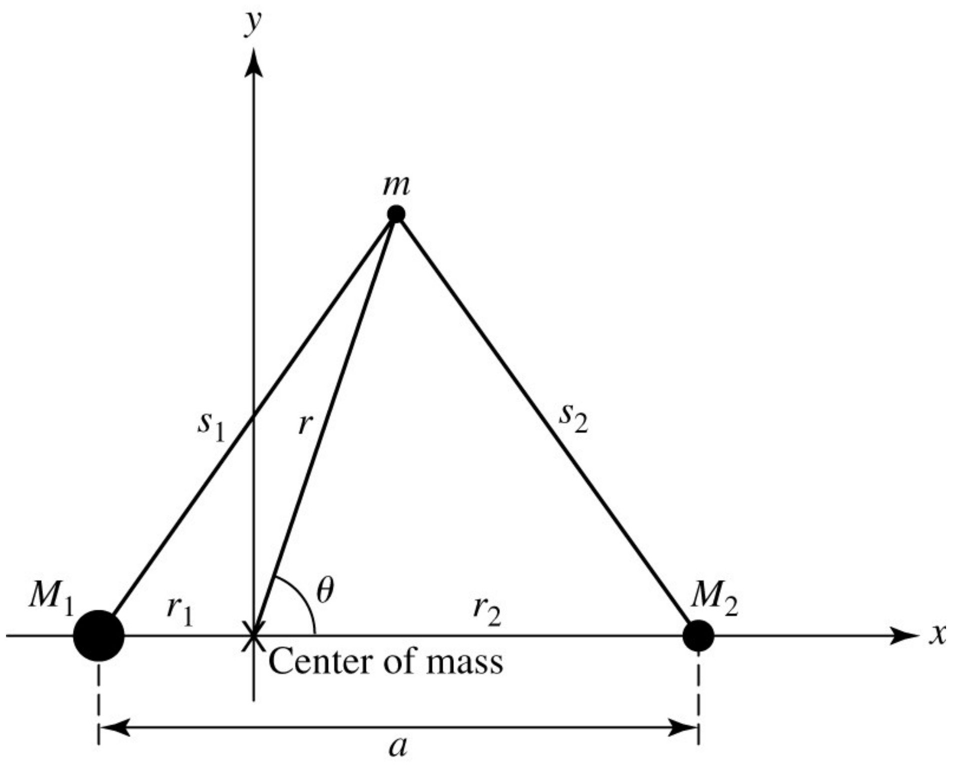
$$U_c = -\frac{1}{2}m\omega^2 r^2$$

Note: your text puts one of the stars at the origin, so the terms are slightly different

Equipotentials

- Center of mass condition: $M_1 r_1 = M_2 r_2$
- Separation: $a = r_1 + r_2$
- Total effective potential energy for a test mass in x-y plane:

$$U = -Gm\left(\frac{M_1}{s_1} + \frac{M_2}{s_2}\right) - \frac{1}{2}m\omega^2 r^2$$



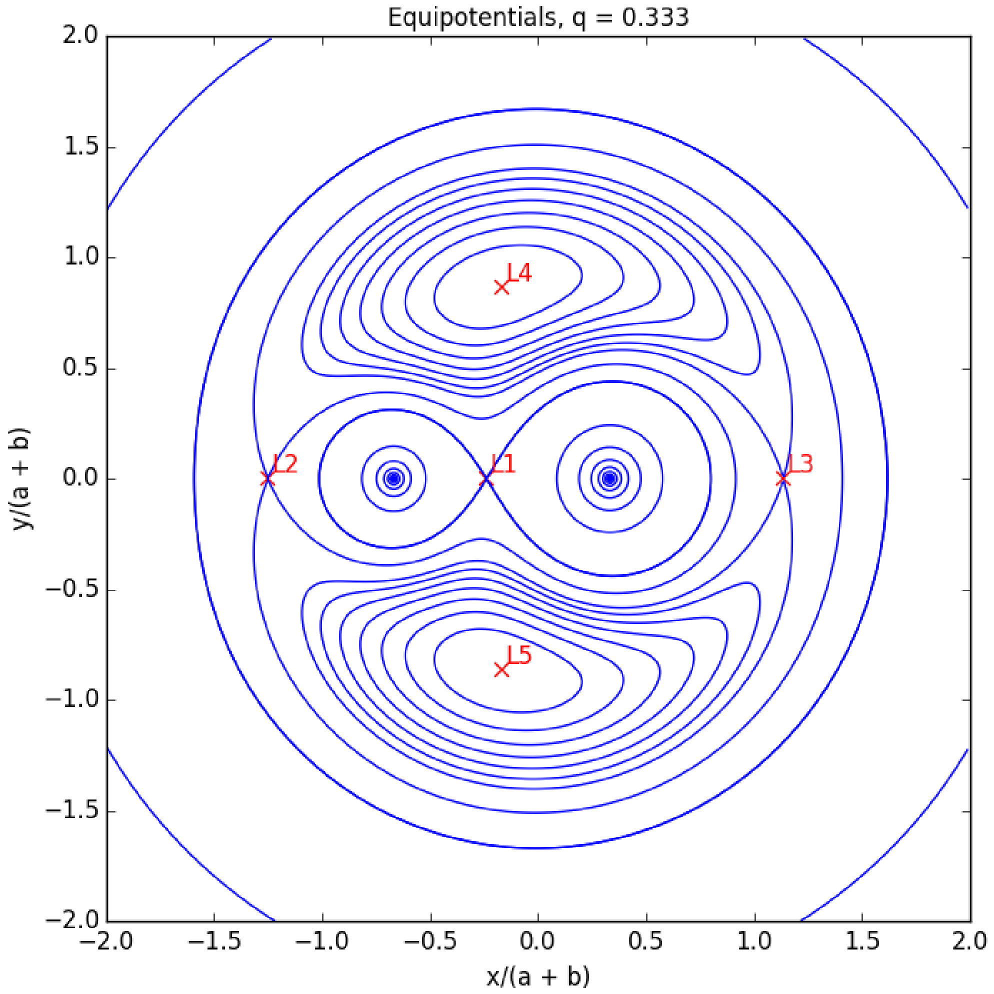
(Carroll & Ostlie)

Equipotentials

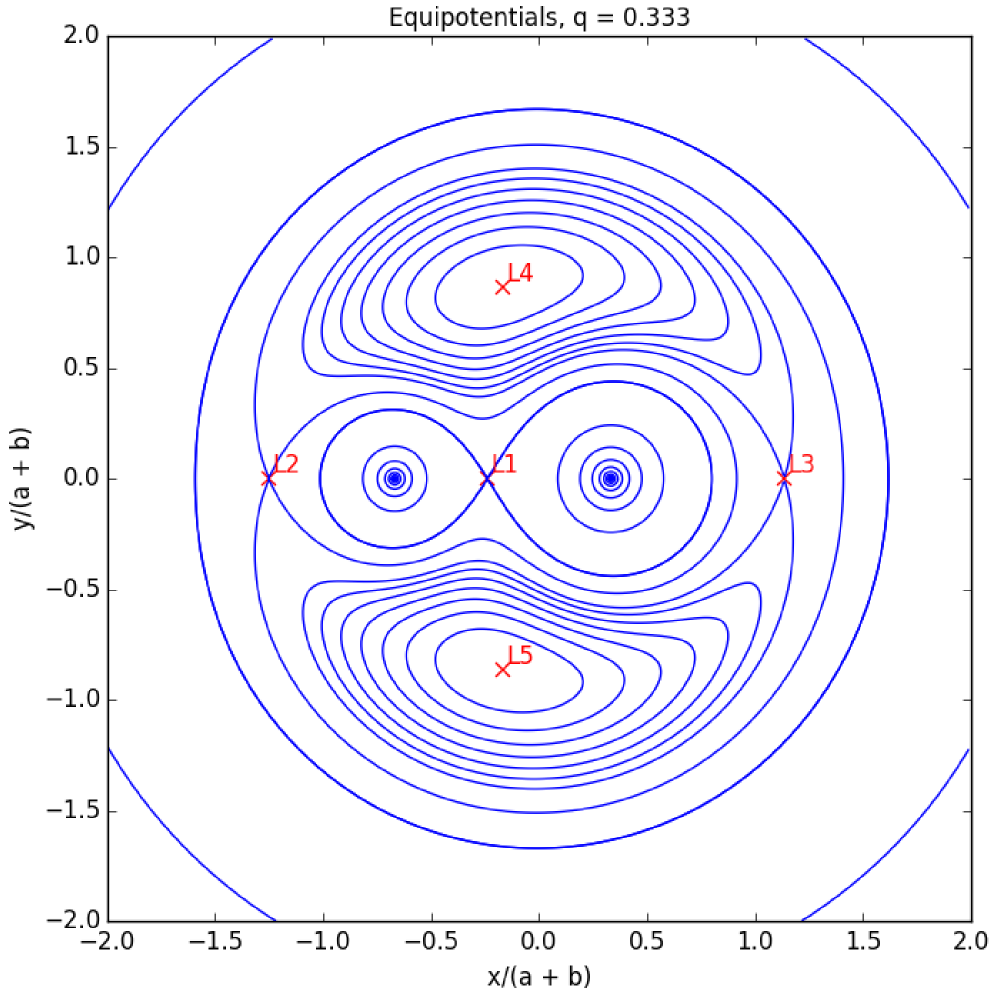
- Define an effective gravitational potential:

$$\Phi = U/m = -G\left(\frac{M_1}{s_1} + \frac{M_2}{s_2}\right) - \frac{1}{2}\omega^2 r^2$$

- 5 Lagrange points
  - On axis:
    - L1 between stars
    - L2 and L3 opposite the stars
    - All unstable
  - L4 and L5
    - Equilateral triangle with masses
    - Equilibrium
    - Trojan asteriods (e.g)



Equipotentials



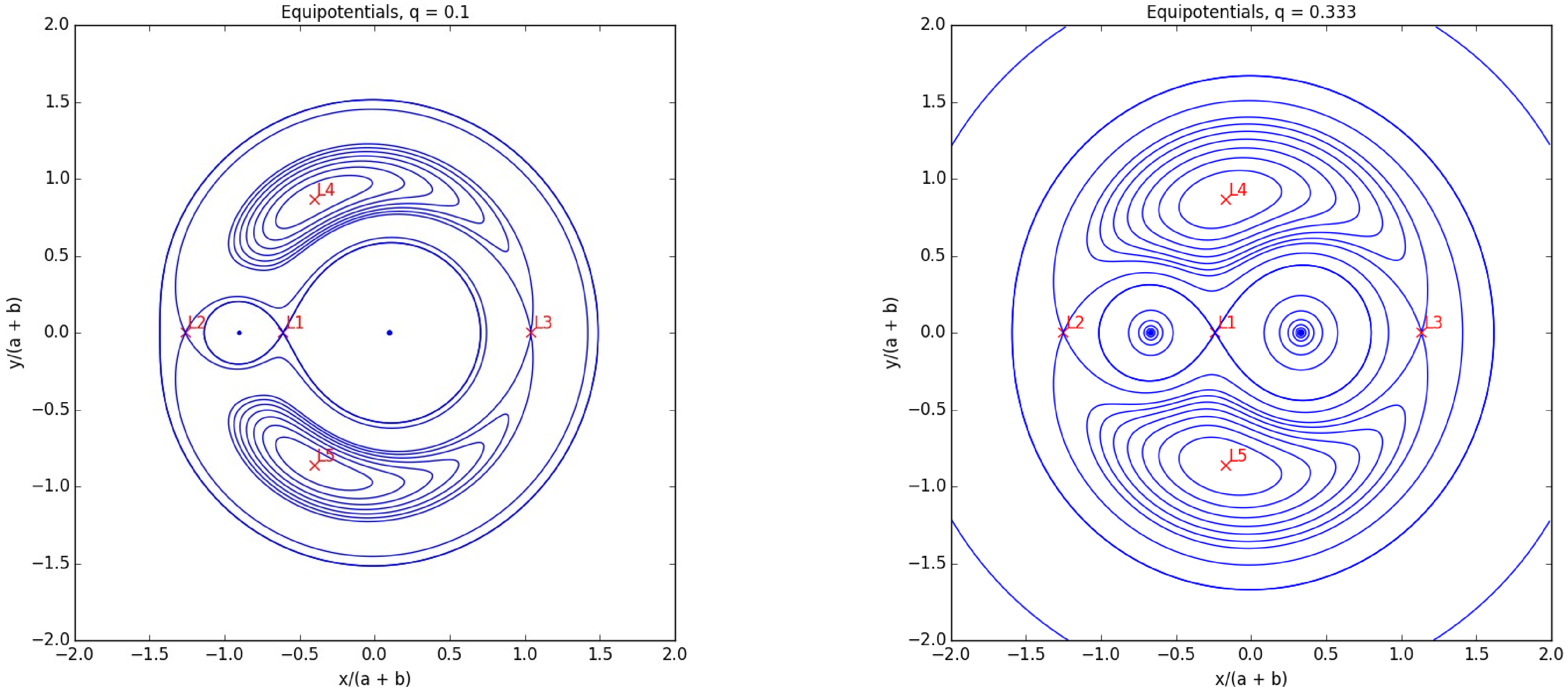
- Dimensionless: distance in units of *a* and mass ratio $q = M_2/M_1$.
  - These plots assume that q < 1
- Roche radius defined as sphere with same volume as Roche lobe
  - Eggleton: least massive lobe is

$$\frac{r_L}{a} = \frac{0.49q^{2/3}}{0.6q^{2/3} + \ln(1 + q^{1/3})}$$

$$\frac{r_L}{a} \sim \frac{1}{2}\left(\frac{q}{1+q}\right)^{1/3}$$

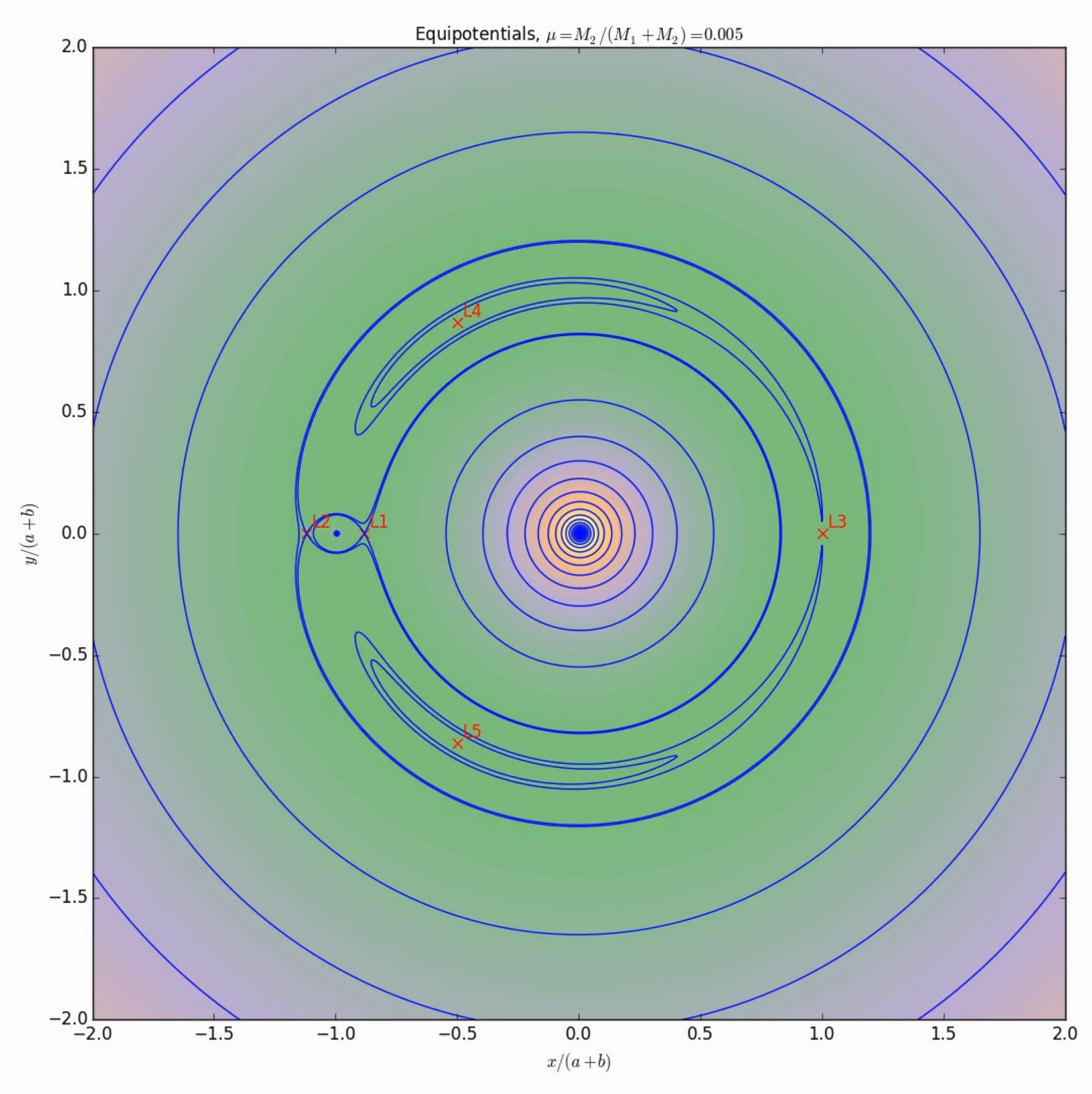
  - more massive lobe: replace *q* with q^{-1}

Equipotentials



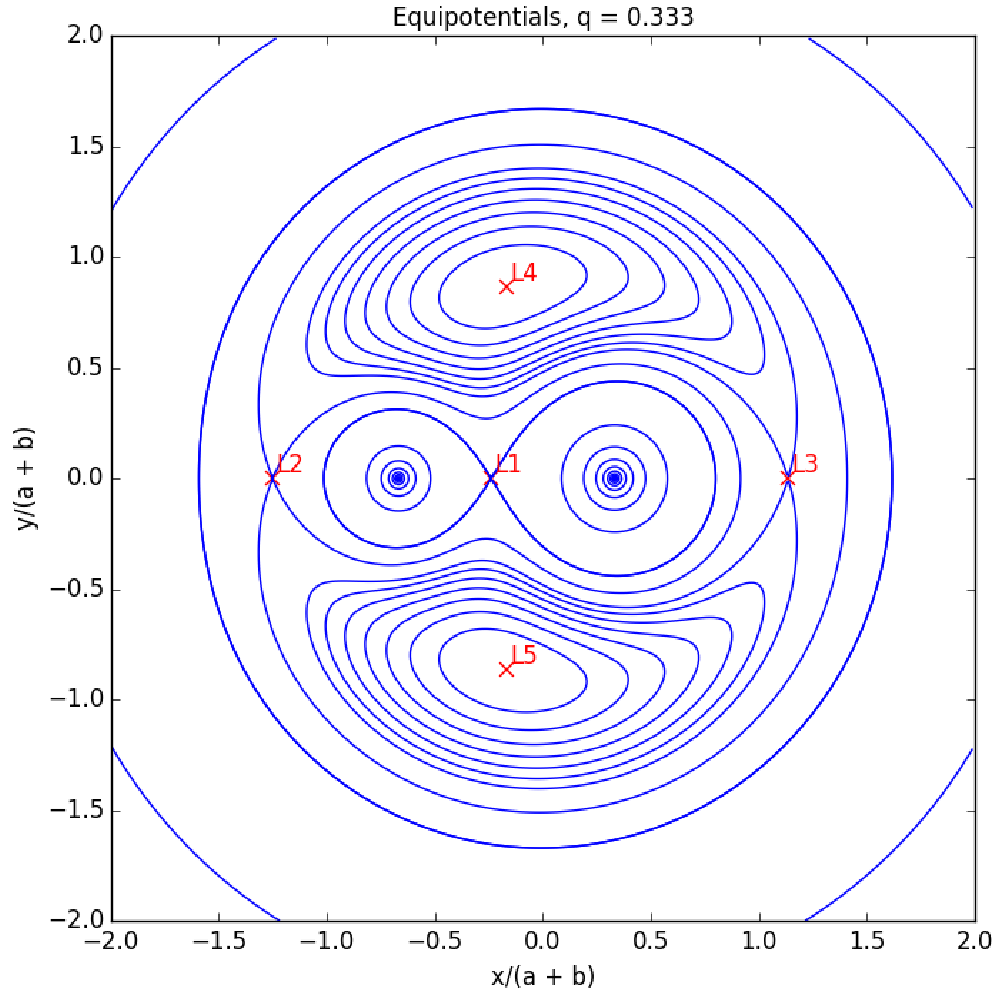
https://github.com/zingale/astro_animations/tree/master/binary_exoplanets/equipotentials

Equipotentials



Equipotentials

- Moving along equipotentials requires no work
  - Effective acceleration normal to equipotentials
- General trends
  - Close to stars:
    - Gravity dominates
    - Equipotentials are spherical; centered on star
  - Far from stars:
    - Centrifugal force domainates
- Roche lobe
  - Each half of the figure-8



Close Binary Systems

- Stars take the shape of equipotentials
  - Horizontal HSE here
    - Gravity is perpendicular to equipotentials, so no acceleration along equipotentials
    - Therefore (HSE) no pressure gradient
    - Density must also be constant, since pressure responds to the weight of the material above it
- Evolved stars in close binary systems will fill their Roche lobes

(Shu)

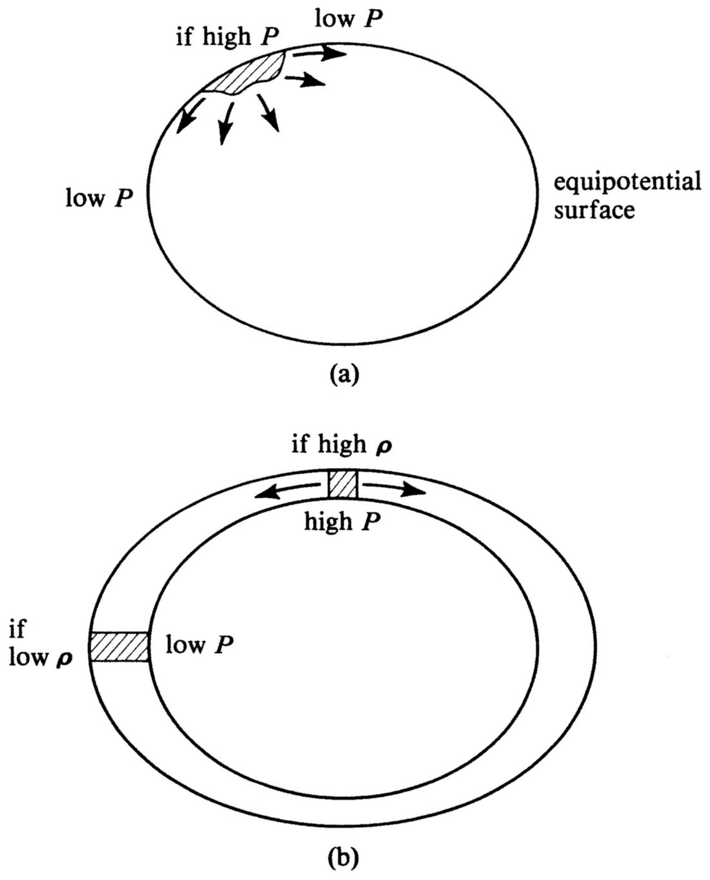
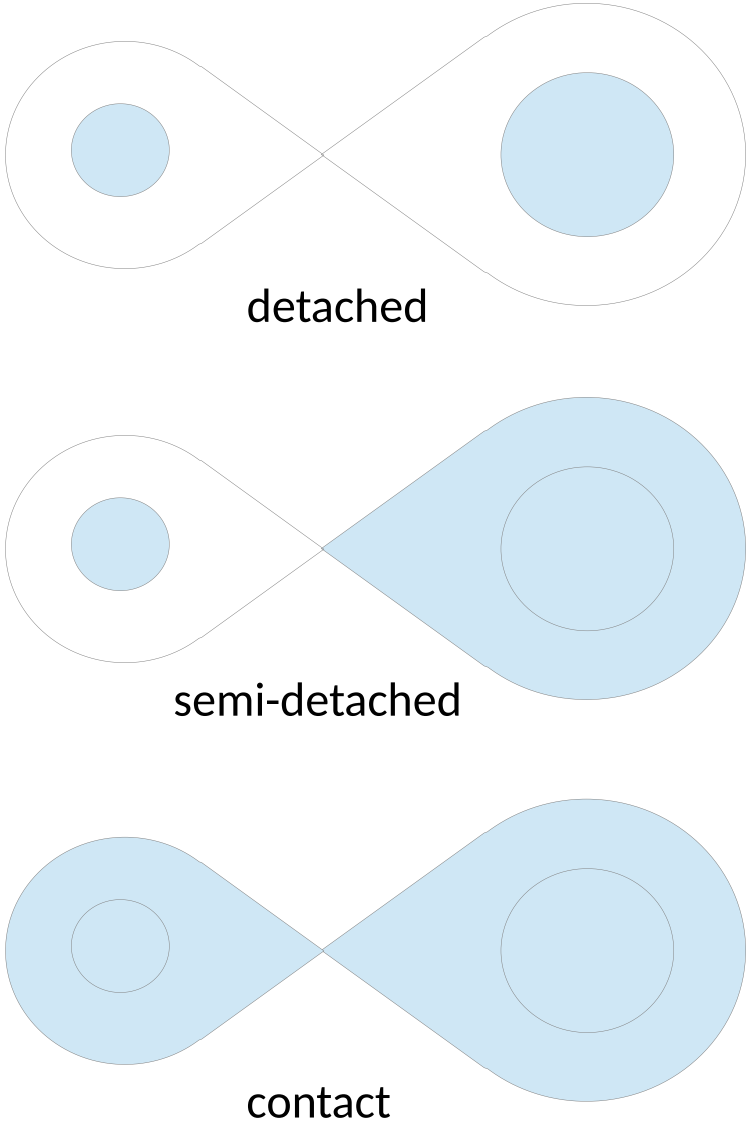


Figure 10.9. (a) Horizontal hydrostatic equilibrium requires the pressure P of a synchronously spinning star to be uniform on an equipotential surface, because an area of high pressure would otherwise be unopposed in its expansion along the surface. (b) Vertical hydrostatic equilibrium requires the density ρ to be uniform, because the weight of a column of high density would otherwise result in a high pressure at its base.

Classification of Close Binaries

- More massive star leaves MS first.
  - R can exceed Roche lobe when red giant
  - Material flows past the L1 point onto companion.
- Binary system classification:
  - Detached: both stars smaller than Roche lobes. Interact via gravity only.
  - Semi-detached: one star fills its Roche lobe. Mass can flow to companion.
  - Contact: both stars fill (or exceed) their Roche lobes. Can have a common envelope surrounding both stars.



Evolution of the System

- What happens to the separation when there is mass transfer?
  - Total angular momentum of system:

$$J = M_a a_1^2 \omega + M_2 a_2^2 \omega + I_1 \omega_1 + I_2 \omega_2$$

  - Orbital angular momentum tends to dominate spin (stars are pretty centrally condensed):

$$J \approx M_1 a_1^2 \omega + M_2 a_2^2 \omega$$

  - Simplifying with center of mass condition:

$$a_1 = a \frac{M_2}{M_1 + M_2} \qquad a_2 = a \frac{M_1}{M_1 + M_2}$$

$$J \approx \frac{M_1 M_2}{M_1 + M_2} a^2 \omega$$

Evolution of the System

- Kepler tells us the orbital frequency:

$$\omega^2 = \frac{G(M_1 + M_2)}{a^3}$$

  - so

$$J \approx \left(\frac{G}{M_1 + M_2}\right)^{1/2} M_1 M_2 a^{1/2}$$

- Now conservation of mass tells us:

$$\frac{d}{dt}(M_1 + M_2) = 0 \quad \rightarrow \quad \dot{M}_1 = -\dot{M}_2$$

- Final evolution:

$$\frac{\dot{a}}{a} = 2\frac{\dot{M}_1}{M_1 M_2}(M_1 - M_2)$$

Evolution of the System

- What does this tell us?

$$\frac{\dot{a}}{a} = 2\frac{\dot{M}_1}{M_1 M_2}(M_1 - M_2)$$

  - If more massive star loses mass, then

$$\dot{M}_1 < 0, \dot{a} < 0$$

    the stars get closer. This can lead to a runaway.
  - If the less massive star loses mass, then

$$\dot{M}_1 > 0, \dot{a} > 0$$

    so they separate
    - but... since M_2 is losing mass, q is getting smaller, so the Roche radius gets smaller, and therefore it becomes easier for M_2 to fill its Roche lobe, ...
    - steady mass transfer can result

Cataclysmic Variables

- Sustained mass transfer (lower mass star losing mass) can lead to cataclysmic variables
  - Outbursts in luminosity from thermonuclear or other energy sources

What Happens Next?

(David A. Hardy & PPARC)

Binary Explosion Taxanomy

- WD systems:
  - classical / recurrant nova: thermonuclear explosion of H layer on surface of WD
  - dwarf nova: instability in the accretion disk that dumps a lot of material onto WD surface at once
  - Type Ia supernova: thermonuclear explosion of an entire WD (or pair)
- NS systems:
  - X-ray burst: thermonuclear explosion of H layer on surface of NS
  - short gamma-ray burst: merger of two NSs
  - binary X-ray pulsar: accretion funneled onto magnetic poles of rapidly rotating NS
- BH systems:
  - accretion onto BH gives rise to X-ray emission (ms timescale rules out NS)

Novae

- References:
  - Gehrz, Truran, Iams, and Starfield, 1998, PASP, 110, 3-26
  - Livio and Truran, 1990, Nonlinear Astrophysics
  - J. R. Buchler and S. T. Gottesman, Ed. Annals of the NY Academy of Sciences v. 617
  - Carroll and Ostlie, Ch. 18
  - Truran, 1982, Essays in Nuclear Astrophysics: William Fowler, p. 467
  - Prialnik, “An Introduction to the Theory of Stellar Structure and Evolution”

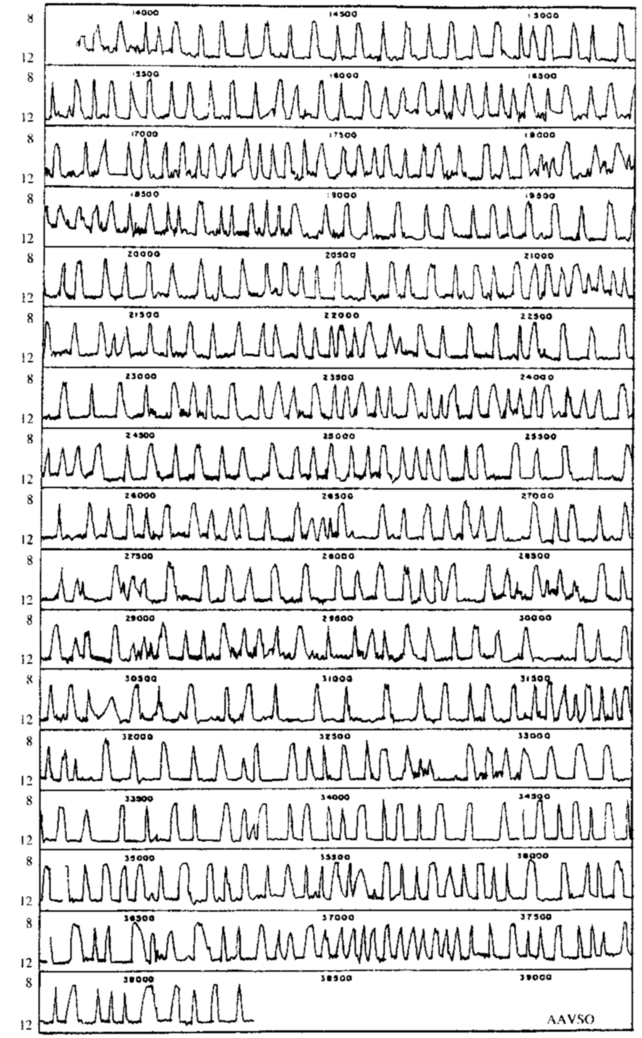
Novae: General Properties

- Classical novae:
  - Increase in brightness by ~ 10^3 to 10^6 ×
  - Remain bright for days to months
  - White dwarf in a binary system (with a low mass main-sequence star as the companion)
  - System is not destroyed by the outburst
- Classical novae never recur over their observed lifetimes (estimated recurrence times of 1000 – 10000 years
- Recurrent novae are related to Classical novae and recur on the timescales of decades

- Dwarf novae:
  - Increase in brightness by ~ 10 ×
  - Also WD in binary system
  - Can recur on timescales of 10s of days

Dwarf Novae

- Outburst is due to increase in mass accretion rate
  - Likely due to a disk instability, where H ionization ($T > 10^4$ K) causes an increase in the viscosity, allowing material to in-fall more easily.
- Only possible for low accretion rates ($\dot{M} < 10^{-11} M_\odot$ / yr)
- Not thermonuclear in nature



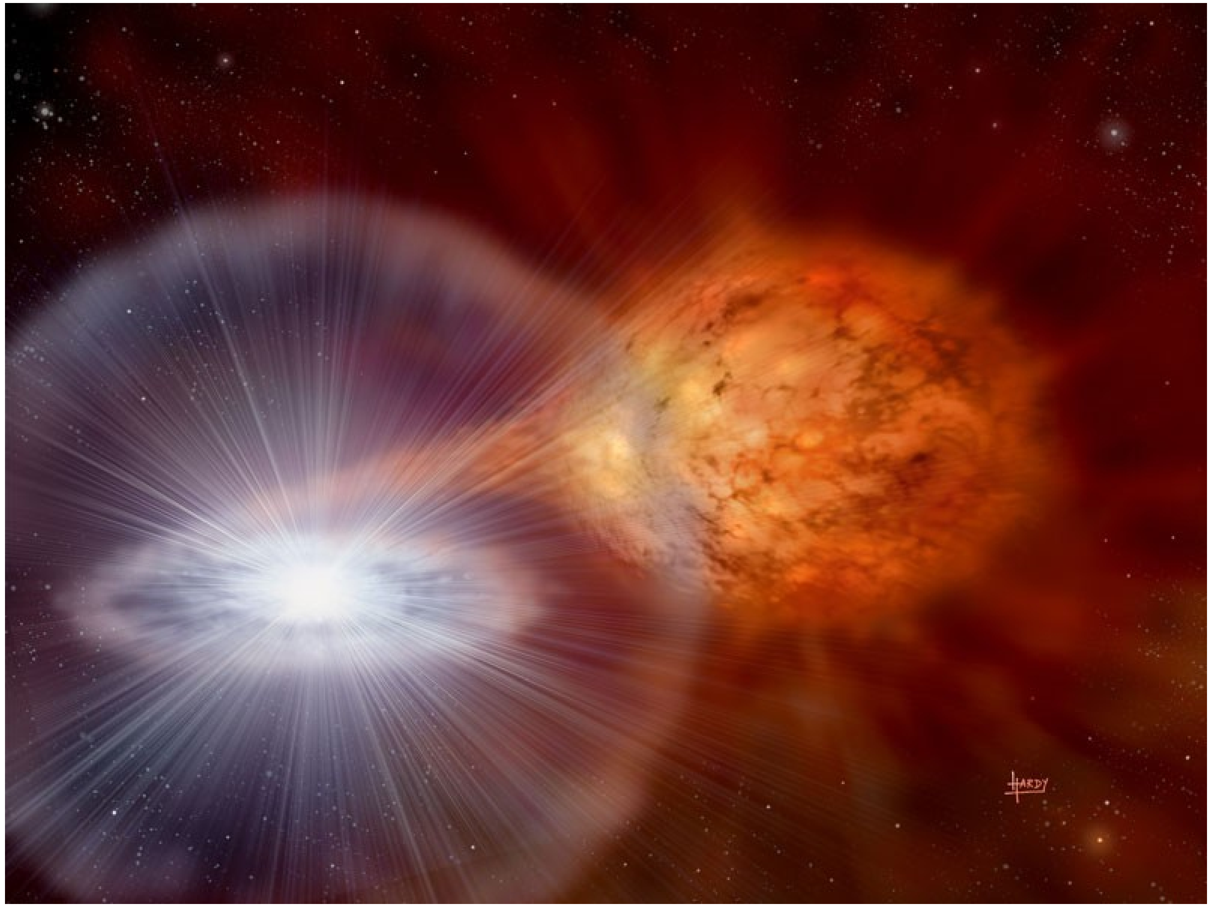
Light curve of SS Cygni
1896–1963

FIGURE 18.16 Outbursts of the dwarf nova SS Cygni, about 95 pc away. This light curve, labeled by Julian day at 500-day intervals, covers the years 1896–1963 and was compiled by the American Association of Variable Star Observers. (We acknowledge with thanks the variable star observations from the AAVSO International Database contributed by observers worldwide.)

(Carroll and Ostlie)

Classical Novae

- Accreted material on surface compressed
- Degeneracy sets in
- T at base reacts H ignition
- Thermonuclear runaway of an accreted H layer on the surface of a white dwarf
- Once it is hot enough, degeneracy lifted, shell expands, and burning is quenched

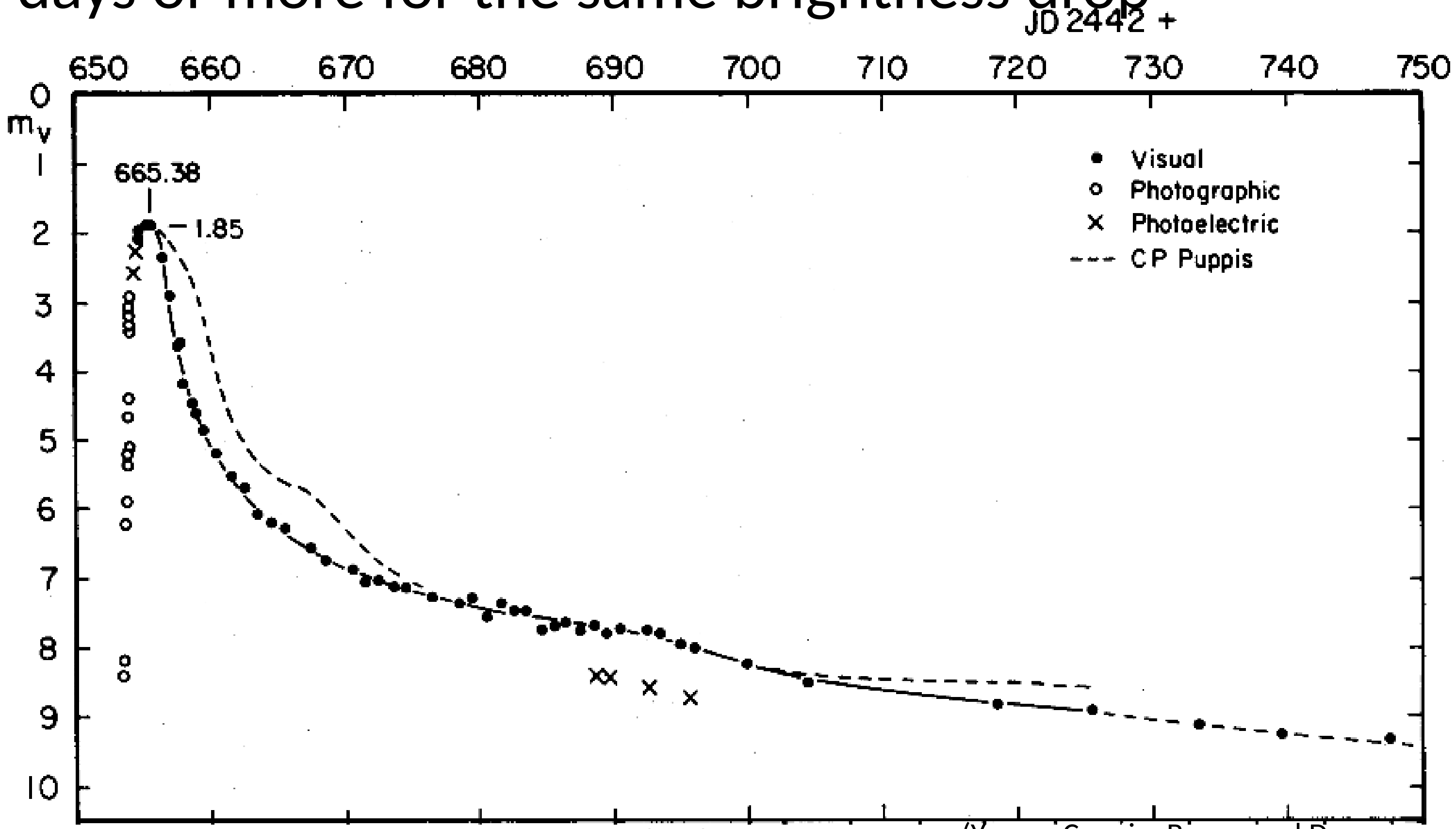


An artists depiction of the RS Ophiuchi nova

(David A. Hardy & PPARC)

Classical Novae

- At peak brightness, nova can have $L \sim 10^5\ L_{\odot}$.
- Fast novae: brightness drops 2 mag in weeks
- Slow novae: 100 days or more for the same brightness drop



(Young, Corwin, Bryan, and De Vaucouleurs)

Classical Novae

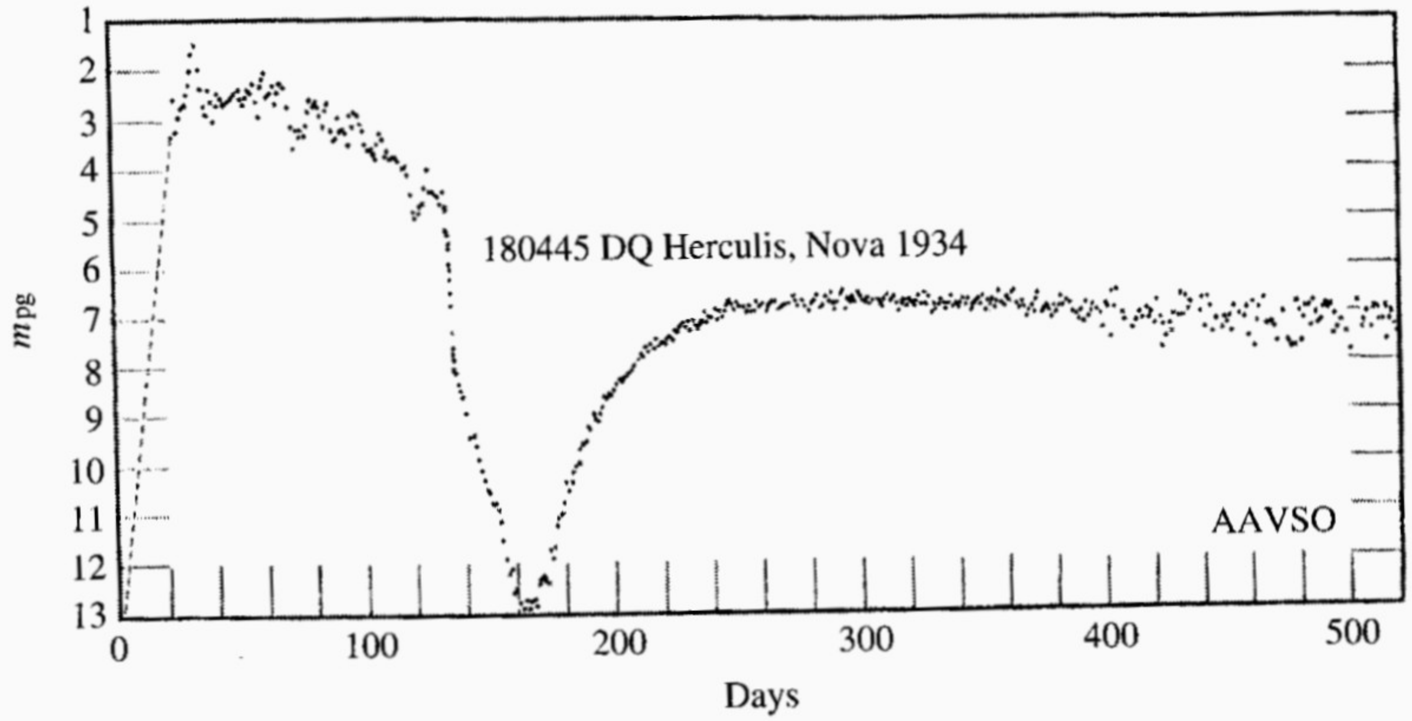
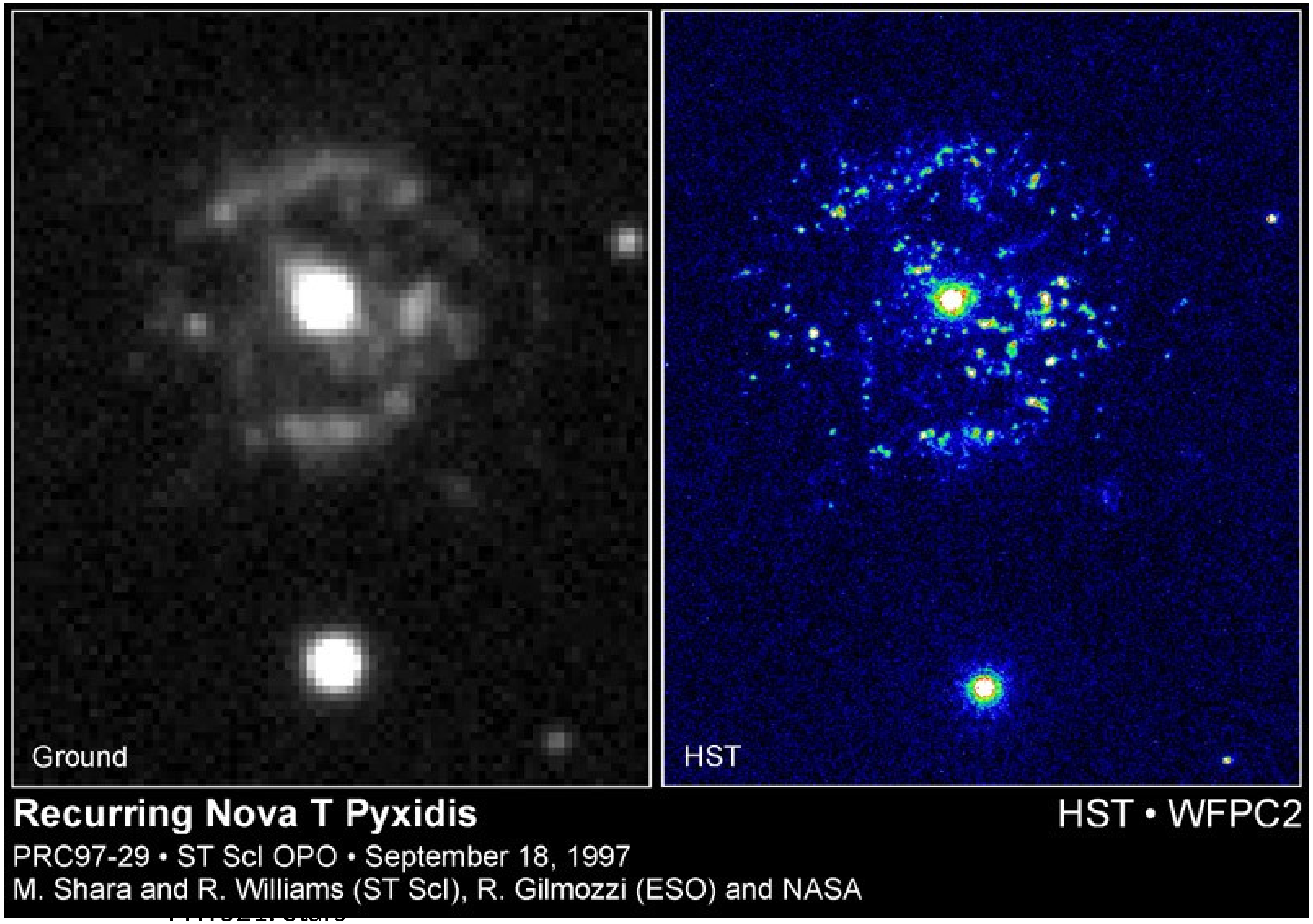


FIGURE 18.18 The light curve of DQ Her, a slow nova. The *photographic magnitude*, m_{pg}, is measured from the nova's image on photographic plates. (We acknowledge with thanks the variable star observations from the AAVSO International Database contributed by observers worldwide.)

(Carroll and Ostlie)

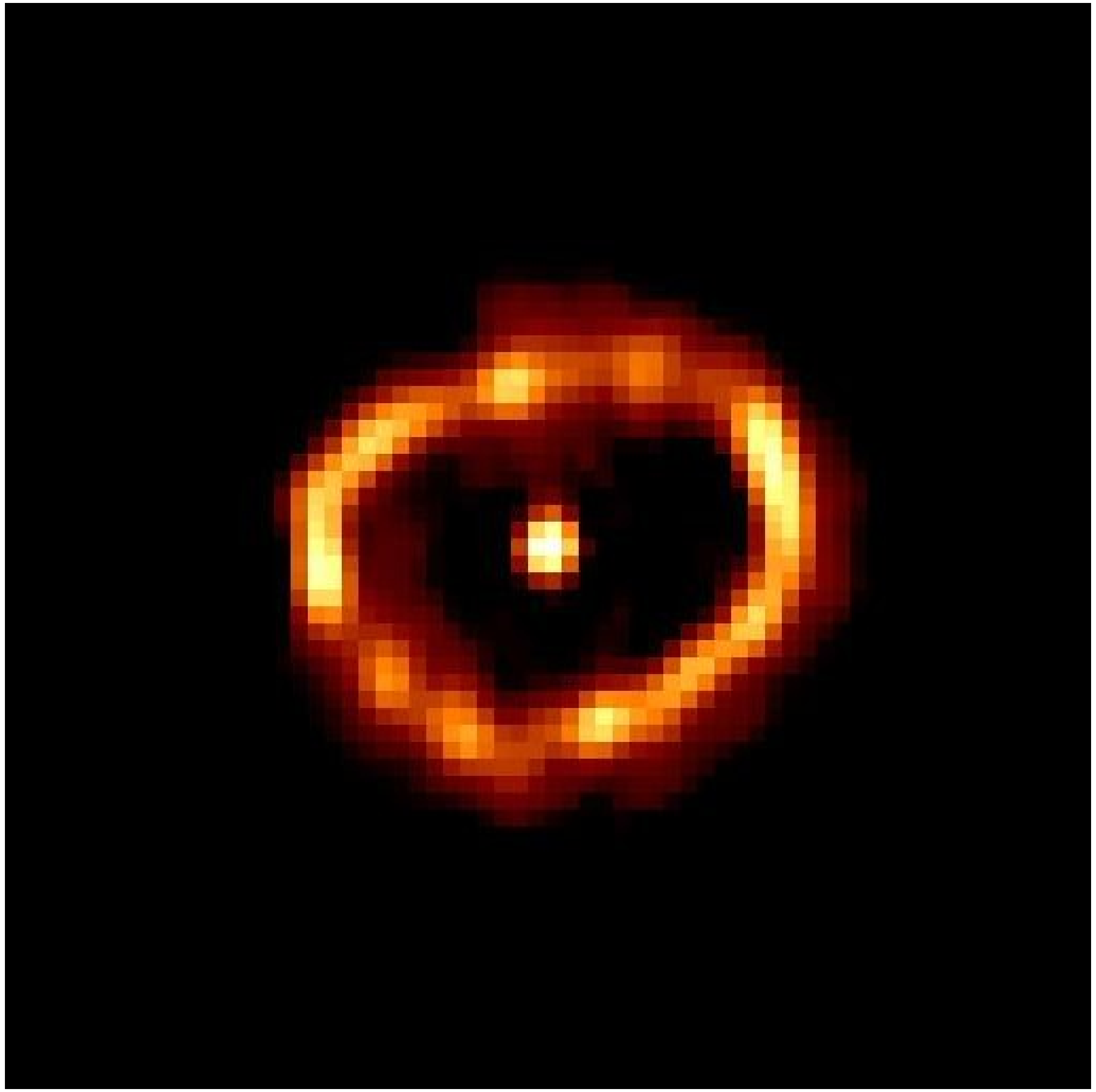
Classical Novae

- ~40 novae / yr in our galaxy
- Some recur (P ~ decades)



Classical Novae

Nova Cygni 1992



(F. Paresce, R. Jedrzejewski (STScI), NASA/ESA)

Classical Novae

- General picture:
  - WD accretes H from companion at 10^{-9} to 10^{-8} $M_{\odot}$ yr^{-1}.
  - H layer builds up
  - Some (poorly-understood) mixing takes place enriching envelope with CNO
  - Conditions at the base are degenerate—runaway!
  - Degeneracy lifted only once T > 10^{8} K

Classical Novae

- When does the runaway occur?
  - Need T > T_{ign} (~ 10^7 K)
- Ignition takes place in deepest layer where H present
  - Conditions will be degenerate

$$\frac{\rho_b k T_{\rm ign}}{\mu_e m_u} \sim K_1' \left(\frac{\rho_b}{\mu_e}\right)^{5/3}$$

- Critical pressure:

$$P_{\rm crit} \sim \left(\frac{k}{m_u}\right)^{5/2} \frac{T_{\rm ign}^{5/2}}{{K_1'}^{3/2}}$$

  - ~ 2 × 10^{18} dyn cm^{-2}

Classical Novae

- HSE:

$$\frac{dP}{dr} = -\rho g$$

$$\frac{0 - P_{\rm crit}}{\Delta r} = -\rho g = -\frac{\Delta M}{4\pi R_{\rm WD}^2 \Delta r} \frac{GM}{R_{\rm WD}^2}$$

$$P_{\rm crit} \sim \frac{GM\Delta M}{4\pi R_{\rm WD}^4}$$

- WD mass-radius relation: $R \propto M^{-1/3}$

$$\Delta M \propto M^{-7/3}$$

Burning

(Prialnik)

- This corresponds to $T \sim 2 \times 10^7$ K
  - CNO cycle dominates the burning
  - Partially degenerate—P response is not great so T increases further
  - Convection sets in
- Above 10^8 K, hot CNO (beta-limited) kicks in
  - β+ decay rates are slow, T independent
  - ^{14}O and ^{15}O build up (these have slow decay times)
  - If we get hot enough, we can break out (rp-process). More on this later (w/ XRBs)
- Degeneracy is lifted here, which can quench the runaway
  - Expansion of the shell, T drops
- Luminosity can be super-Eddington

- Hot CNO

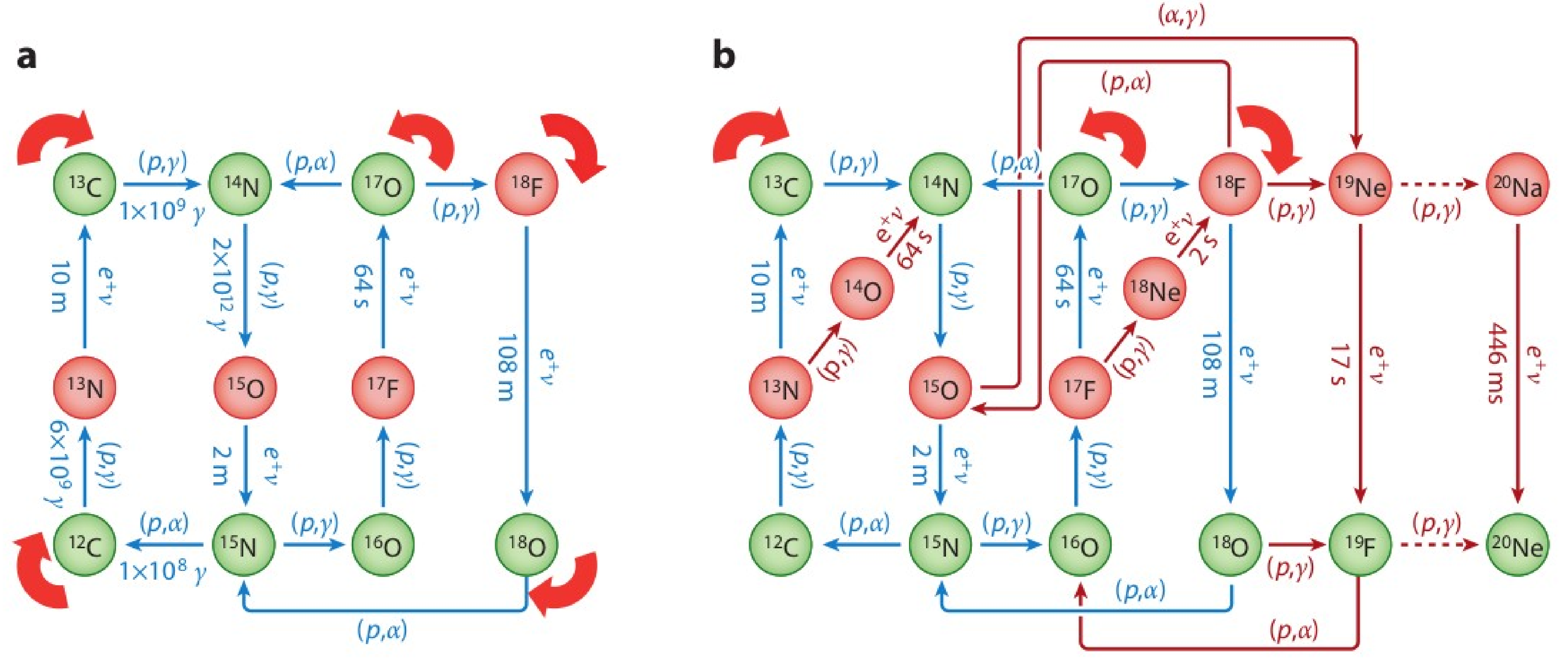


Figure 1

(*a*) The reaction flow in the three cold CNO cycles. (*b*) The hot CNO cycles, which represent a much more complex cyclic reaction pattern. Dashed lines indicate breakout reactions into the NeNa mass range, which occur at temperatures in excess of 0.4 GK.

(Wiescher et al. 2010)

Classical Novae

- From Gehrz et al:

TABLE 1

CLASSICAL NOVA WHITE DWARF AND OUTBURST CHARACTERISTICS

M_{WD} ($M_\odot$)	R_{WD} (cm)	M_{envl} ($M_\odot$)	ε_b (ergs g^{-1})	E_b (ergs)	L_{Pacz} ($L_\odot$)	L_{Edd} ($L_\odot$)	$\dot{M}_{nucl}$ ($M_\odot$ yr^{-1})	τ_{nucl} (yr)	τ_{recur} (yr)	f
0.6	9.5 (8)	1.3 (−3)	8.4 (16)	2.2 (47)	4.62 (3)	2.28 (4)	6.44 (−8)	2.02 (4)	1.3 (6)	0.100
0.7	8.6 (8)	7.3 (−4)	1.1 (17)	1.6 (47)	1.05 (4)	2.66 (4)	1.47 (−7)	4.98 (3)	7.3 (5)	0.053
0.8	7.7 (8)	4.2 (−4)	1.4 (17)	1.2 (47)	1.64 (4)	3.04 (4)	2.29 (−7)	1.83 (3)	4.2 (5)	0.042
0.9	6.9 (8)	2.4 (−4)	1.7 (17)	8.4 (46)	2.24 (4)	3.42 (4)	3.13 (−7)	770	2.4 (5)	0.040
1.0	6.1 (8)	1.3 (−4)	2.2 (17)	5.7 (46)	2.83 (4)	3.80 (4)	3.95 (−7)	330	1.2 (5)	0.046
1.1	5.2 (8)	6.4 (−5)	2.8 (17)	3.6 (46)	3.42 (4)	4.18 (4)	4.77 (−7)	130	6.4 (4)	0.062
1.2	4.4 (8)	2.8 (−5)	3.6 (17)	2.1 (46)	4.02 (4)	4.56 (4)	5.61 (−7)	50	2.8 (4)	0.100
1.3	3.3 (8)	9.0 (−6)	5.3 (17)	9.4 (45)	4.61 (4)	4.94 (4)	6.43 (−7)	14	9.0 (3)	0.230
1.35	2.7 (8)	4.0 (−6)	6.7 (17)	5.3 (45)	4.91 (4)	5.13 (4)	6.85 (−7)	5.6	4.0 (3)	0.320

  - Nova properties assuming $\dot{M} = 10^{-9}\ M_\odot\ \mathrm{yr}^{-1}$
  - Higher mass WDs reach ignition with less massive envelopes
    - For a given accretion rate, they occur more frequently

Classical Novae

(Truran 1982; Carroll and Ostlie, Ch. 18)

- H burning makes ~ 6 $\times 10^{18}$ erg g^{-1}
  - 10^{-4} $M_\odot$ envelop releases 10^{48} erg $\gg$ $E_{binding}$
  - Observed integrated luminosity is 10^{46} erg
- Observed luminosity → small fraction of the envelop is burned, or most of the mass is lost (ejected) during the event.
  - Velocities far from explosion are small—material may just escape
  - About 10% of H is ejected during explosion
- Hydrostatic burning follows
  - Layer above burning shell expands, becomes convective
  - May overflow Roche lobe
  - Burning stops when remaining material ejected

Classical Novae

(Gehrz et al.)

- About 10^{-4} $M_\odot$ is ejected to the ISM
- ~40 novae / yr in the galaxy
  - Total ejected masses is ~4 x 10^{-3} $M_\odot$ yr^{-1}
- Supernovae occur about 1 every 50 yr, but eject ~3 $M_\odot$ per event, or 0.06 $M_\odot$ yr^{-1}
- Novae are a small part of nucleosynthesis
  - Except for the elements they overproduce

Modeling Novae

- Modeling novae is hard because of the large amount of expansion in the envelope
  - Late stages, $R \sim 10^{12}$ cm
- Major theoretical issue: dredge-up
  - How to we enrich the burning layer with CNO from the underlying WD?
  - Algorithmic issues may mask physical effects here

Modeling Novae

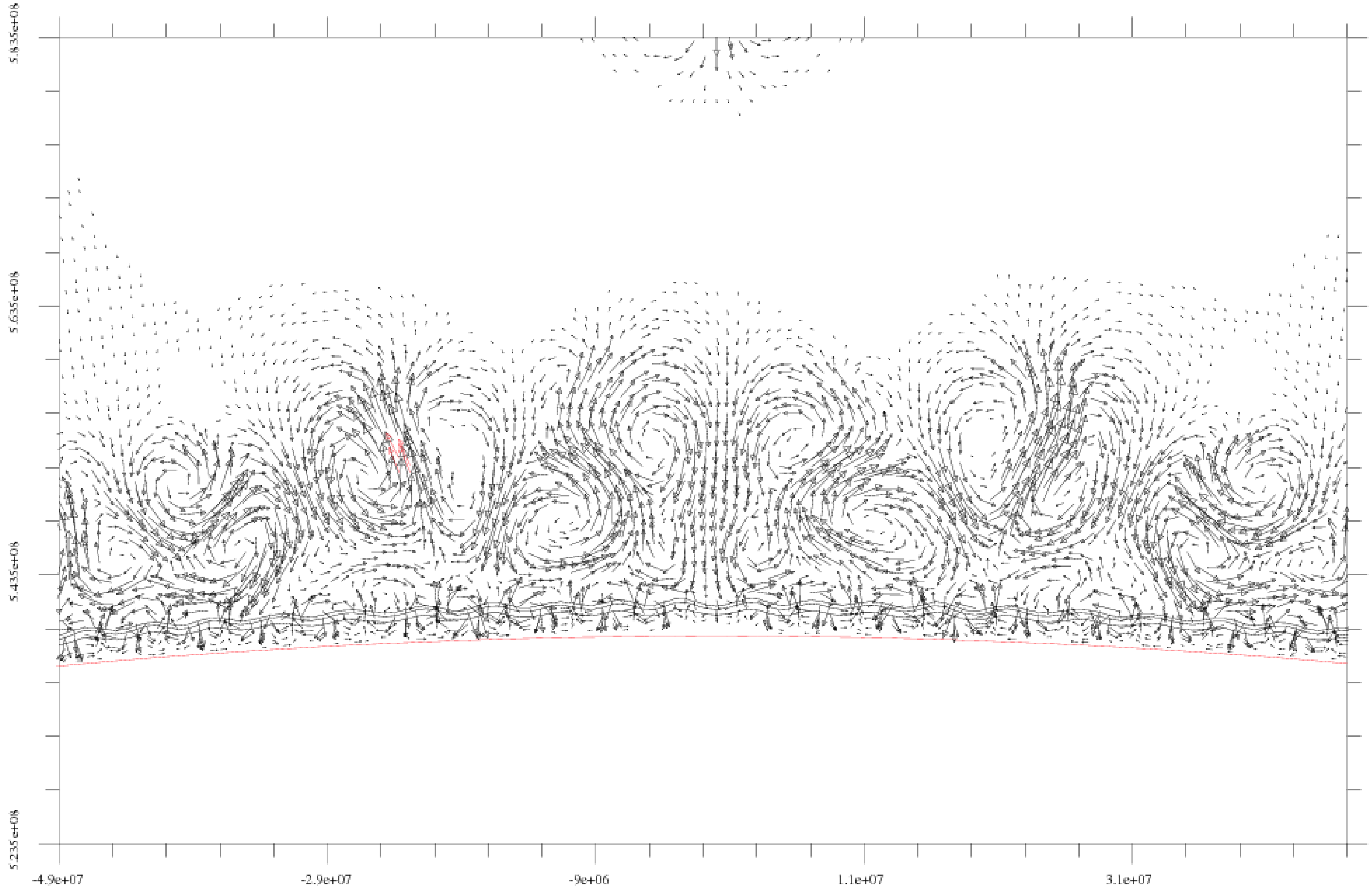


FIG. 2.—Snapshot of the early Kelvin-Helmholtz instability at t = 30 s. Contour lines present the abundance of ^{12}C in the interval [0.001–0.5].

Nova Dredge-Up

(Livio & Truran)

- How can we dredge-up material from the underlying WD?
  - Diffusion layer
    - H diffuses down into WD during accretion
    - Deep H ignites first, with lots of metals surrounding it
    - Convection driven by this heating brings metals into the H envelope
  - Shear mixing
    - Accretion disk extends to the WD surface
    - Kelvin-Helmholtz instability ensues and mixed
    - Can this work with a magnetic field?
  - Convective overshoot
    - Burning begins at the base of the H layer
    - Convection is driven
    - Overshoot of the convective eddies mixed CO into the envelope

Nova Dredge-Up

- Another possibility is gravity waves (like in the ocean)

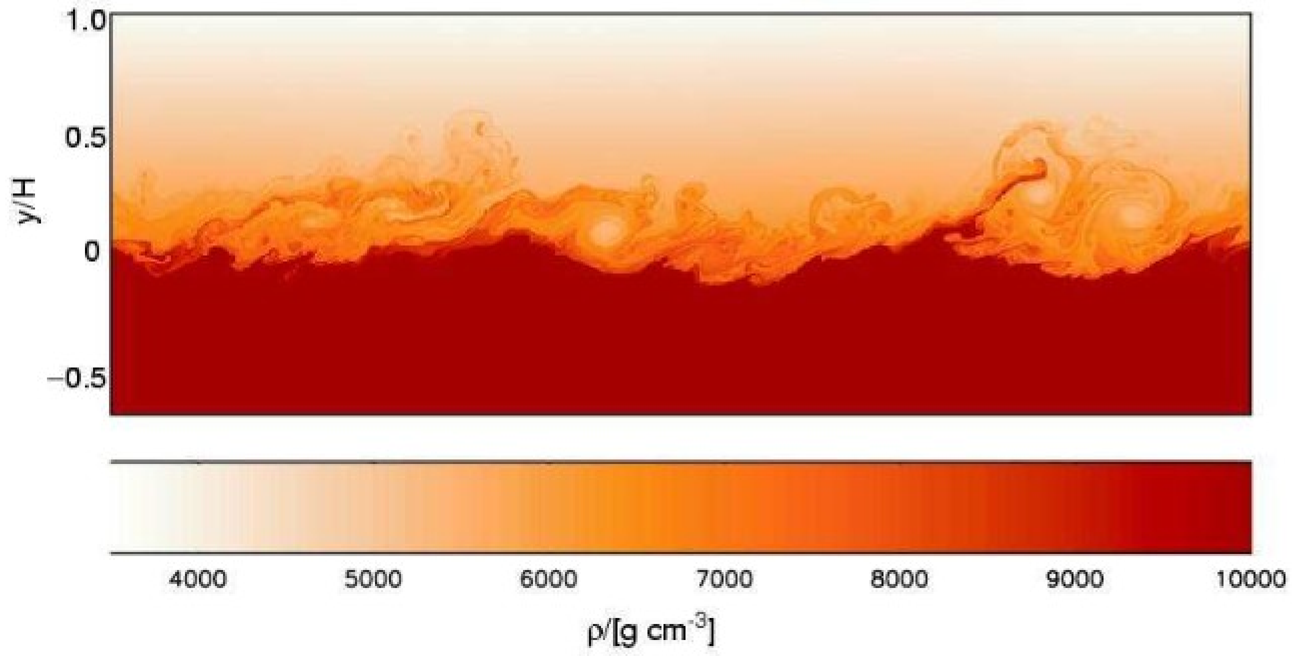


FIG. 1.— Breaking CO waves, as determined by simulations in two dimensions. Gravity points towards the bottom of the figure, with the vertical distance y in units of the pressure scale height H, as evaluated just above the interface. The color scale indicates the mass density in units of $\mathrm{g\,cm^{-3}}$.

(Alexakis et al.)

Novae: Recent Simulations

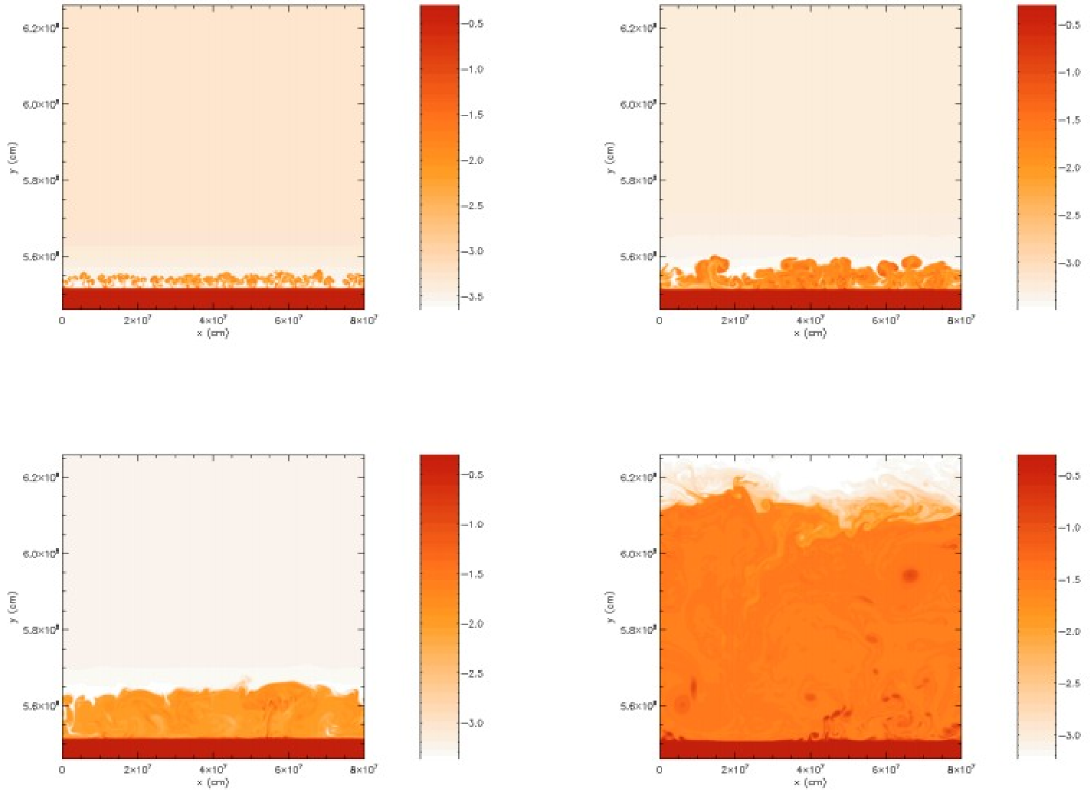


Fig. 1. Snapshots of the development of KH instabilities at $t = 215$ s (*upper left panel*), 235 s (*upper right*), 279 s (*lower left*), and 498 s (*lower right*), shown in terms of ^{12}C mass fraction (in logarithmic scale). The injection of core material driven by the KH instabilities translates into a mass-averaged abundance of CNO-nuclei in the envelope of 0.079, 0.082, 0.089, and 0.17, respectively. The mean CNO abundance at the end of the simulations reaches 0.30, by mass.

(Casanova et al. 2010)

Novae: Outstanding Problems

(taken from S. Starfield, “Studies of Nove in the 20th Century”, in Classical Nova Explosions, M. Hernanz and J. Jose, ed. 2002, AIP Press)

- Mass accretion rates that are observed don't result in nova explosions when run numerically (observed rates seem too high)
- Lots of core material present in ejecta—how's it get there
- More material is ejected in observed systems than models predict
- Do they make ^{26}Al?
- Are recurrent nova systems the progenitors of SNe Ia?

Nova News

- Recent results in Nova observations include:
  - Gamma-rays associated with several novae (see e.g. Hernanz, http://arxiv.org/abs/1305.0769)
    - perhaps there is a late-time shock?
  - Recurrant nova in M31 with recurrance time < 1 year (see Darnley et al., http://arxiv.org/abs/1506.04202)
    - Must be massive—can it be a Ia progenitor?
    - Is it CO or ONeMg?
  - Maybe WDs undergoing novae actually gain in mass over time (see Zorotovic et al., http://adsabs.harvard.edu/abs/2011A%26A...536A..42Z)
    - Compared pre-CV WD masses to CV WD masses, and found that WDs that exhibit CVs are significantly more massive—implying that they grow in mass over time.

Thin Shell Instability

(following Glatzmaier notes and Prialnik Ch. 6)

- Integrating inward (P vanishes at surface):

$$P_{\rm sh} = -\int_{m_{\rm sh}}^{M} \frac{Gm}{4\pi r^4} dm$$

  - m_{sh} is the mass of star interior to shell
- Perturb pressure by δP:

$$P_{\rm sh} + \delta P \sim -\int_{m_{\rm sh}}^{M} \frac{Gm}{4\pi \left[r(1+\delta r/r_{\rm sh})\right]^4} dm$$

  - Use homologous expansion to capture the expansion of the outer edge of the shell

- Assume a small perturbation:

$$P_{\rm sh} + \delta P \sim \left[1 - 4\frac{\delta r}{r_{\rm sh}}\right] P_{\rm sh}$$

  - or

$$\frac{\delta P}{P_{\rm sh}} \sim -4\frac{\delta r}{r_{\rm sh}}$$

Thin Shell Instability

- What about density?

$$\rho = \frac{\Delta m}{4\pi r_{\rm sh}^2 l}$$

  - perturbing:

$$\rho + \delta\rho \sim \frac{\Delta m}{4\pi r_{\rm sh}^2 (l + \delta r)} \sim \rho \left(1 - \frac{\delta r}{l}\right)$$

$$\therefore \qquad \frac{\delta\rho}{\rho} = -\frac{\delta r}{l} = -\frac{\delta r}{r_{\rm sh}} \frac{r_{\rm sh}}{l}$$

- Together:

$$\frac{\delta P}{P_{\rm sh}} \sim 4 \frac{\delta\rho}{\rho} \frac{l}{r_{\rm sh}}$$

  - now, since $l \ll r_{sh}$, the right side is very small
  - This equation tells us that for a given change in density, the fraction cange in pressure is much smaller

Thin Shell Instability

- Now consider our EOS in the form:

$$P \propto \rho^a T^b$$

  - So

$$\frac{dP}{P} = a\frac{d\rho}{\rho} + b\frac{dT}{T}$$

- Everything together:

$$\frac{\delta P}{P} = 4\frac{l}{r_{\rm sh}}\frac{\delta\rho}{\rho}$$

$$\left(4\frac{l}{r_{\rm sh}} - a\right)\frac{\delta\rho}{\rho} = b\frac{\delta T}{T}$$

  - Thermal stability requires that expansion leads to a T drop
    - a and b are positive for general EOS

$$4\frac{l}{r} > a$$

  - A shell can be very thin, and therefore unstable
    - very thin shell—temperature increases as the shell expands!
    - instability can exist until the shell becomes think, allowing for a temperature drop

Other Instabilities

- Thin shell instability was first worked out by Schwarzschild and Härm (1965)

- A similar perturbative analysis can show that an ideal gas is stable (expansion leads to a temperature drop), but a degenerate gas is unstable
  - Degenerate gas → P not very T sensitive
  - Dump energy into star → T increases → reactions proceed more vigorously (very T sensitive) → T increases further
  - In a normal star, the T increase will lead to a P increase, and the star would expand, quenching the reactions
  - In a degenerate star, the T increase doesn't change P, so the star does not response. Enormous amounts of energy dumped into star → explosion.

Neutron Star Systems

- What about a system with a neutron star as the compact object?
  - Can we form such a system?
- A lot of energy is released in a core-collapse supernova.
  - The neutron star that is formed can be given a strong kick
  - It may be difficult for it to stay bound to its companion.
- Explosion of a massive star (10s of $M_\odot$) leaves behind a neutron star remnant of < a few solar masses
- Explosion drives away about ½ of the mass of the system.
  - Only some systems will remain bound.
- If enough mass is transferred from the massive star to the companion during evolution, the system can remain bound.
  - Or a white dwarf collapses into a neutron star.
- Surprisingly, double neutron star systems are observed!

Neutron Star Systems

- X-ray pulsar:
  - Companion overflows Roche lobe → accretion disk forms → material spirals onto the neutron star.
  - Strong magnetic fields can disrupt the disk—material funneled onto magnetic poles.
  - Gravitational potential energy released, which is radiated away.
  - The neutron star emits X-rays around the magnetic poles.
- The emitting region can be periodically eclipsed, resulting in a binary X-ray pulsar.

Neutron Star Systems

- Her X-1: brightness fluctuations in X-ray emission are seen with period of 1.24 s.
- This is too fast for a white dwarf to be the source.

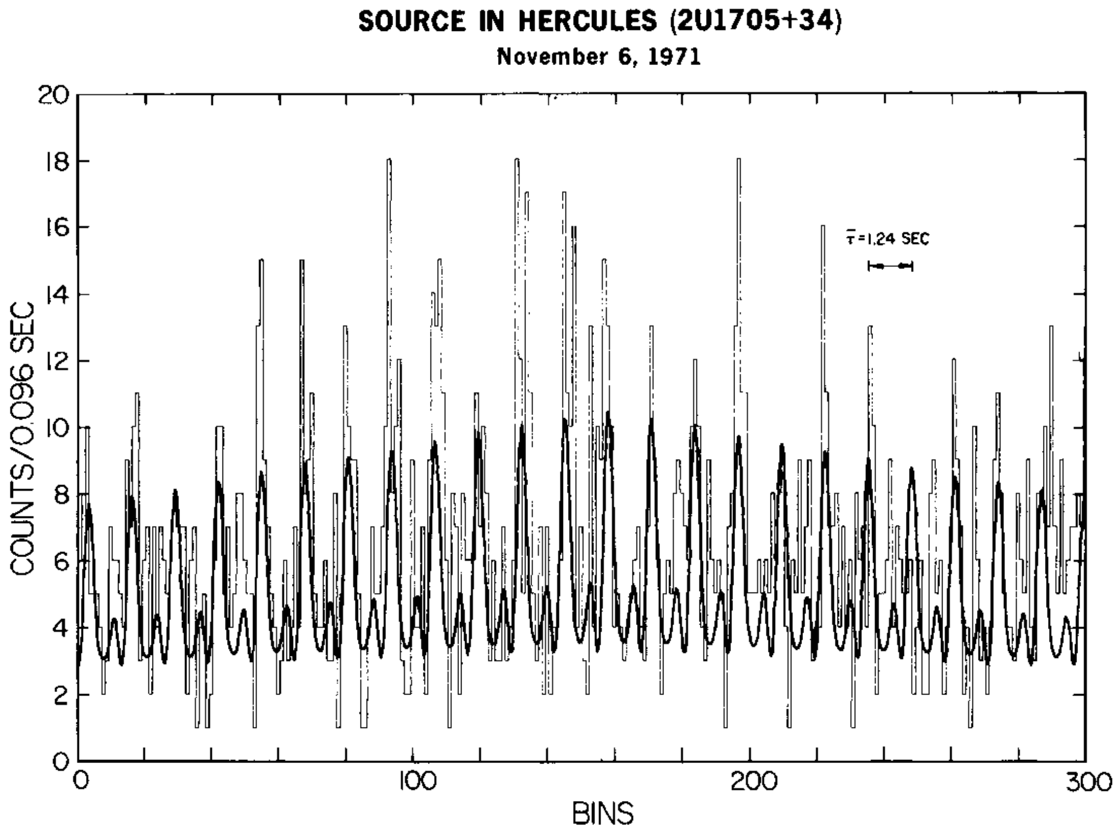


FIG. 1.—The counts accumulated in 0.096-second bins from Hercules X-1 during the central 30 seconds of a 100-second pass on 1971 November 6. The heavier curve is a minimum χ^2 fit to the pulsations of a sine function, its first and second harmonics plus a constant, modulated by the triangular response of the collimator. The functional fit is systematically below the peak counting rate partly due to the sharpness of the pulsing and partly due to the minimum χ^2 technique.

(Tananbaum et al.)

X-ray Bursts

- References
  - Strohmayer and Bildsten 2003, “New Views of Thermonuclear Bursts”
  - Bildsten 2000, “Theory and Observation of Type I X-Ray Bursts from Neutron Stars”

XRBs

- Neutron stars also can accrete H/He from companion
  - Much higher surface g
  - Only meters needed for runaway
  - X-ray burst: explosive burning gives X-ray flash (minutes long)
  - Recurrence time of hours
- Satellites can see repeated bursts from a single source

XRB Energetics

- Gravitational energy release / baryon:

$$\frac{GMm_p}{R} = \frac{(6.67 \times 10^{-8}\ \mathrm{cm}^3\ \mathrm{g}^{-1}\ \mathrm{s}^{-2})(1.4 \cdot 2 \times 10^{33}\ \mathrm{g})(1.67 \times 10^{-24}\ \mathrm{g})}{10^6\ \mathrm{cm}}$$

$$= 3.1 \times 10^{-4}\ \mathrm{erg} \approx 200\ \mathrm{MeV}$$

- Thermonuclear burning (H) releases ~ 5 MeV / baryon
- Burning is overwhelmed by accretion
  - Fuel must be stored and then burned on short timescale
- Thin-shell instability
  - Fuel accreted for hours – days
  - Burned in 10 – 100 seconds
- ~70 XRB sources known (some with > 100 individual bursts)

XRB Burning Regimes

- Solar-like material accretes onto the surface
- Ignition occurs when critical P reached (10^{22} – 10^{23} erg/cm^3)
- Column accretion rate determines what fuel ignites first (see Bildsten 2000)

$\dot{m} < 900 \text{ g cm}^{-2} \text{ s}^{-1}$	Mixed H/He burning, starts with H ignition
$\dot{m} > 900 \text{ g cm}^{-2} \text{ s}^{-1}$ $\dot{m} < (2-5) \times 10^3 \text{ g cm}^{-2} \text{ s}^{-1}$	Pure He ignition (H stably burns to He first). He burst
$\dot{m} > (2-5) \times 10^3 \text{ g cm}^{-2} \text{ s}^{-1}$	Mixed H/He burning, starts with He ignition

X-ray Burst Conditions

- He burst: the conditions at the base of the accreted layer are

$$\rho = 2 \times 10^6 \text{ g cm}^{-3} \qquad T = 1.5 \times 10^8 \text{ K}$$

- Consider the ions / electrons at the base.
  - Begin by treating everything as an ideal gas, and taking the envelope to be He.
  - Charge neutrality: $n_e = 2n_I$ (He nucleus has 2 protons)

$$P_{\text{tot}} = P_e + P_i = n_e kT + n_i kT = 3n_i kT$$

- The mass density is

$$\rho = 4m_p n_i + m_e n_e \approx 4m_p n_i$$

- so

$$P_{\text{tot}} = 3\frac{\rho}{4m_p}kT$$

X-ray Burst Conditions

- Using the conditions at the base, we have

$$P_{\rm tot} = 3\frac{2\times 10^{6}\ {\rm g\ cm^{-3}}}{4\cdot 1.67\times 10^{-24}\ {\rm g}} 1.38\times 10^{-16}\ {\rm erg\ K^{-1}}\cdot 1.5\times 10^{8}\ {\rm K}$$
$$= 1.8\times 10^{22}\ {\rm erg\ cm^{-3}}$$

- Note that $P_e = 2P_i = 2P_{\rm tot}/3$
  - If we instead treat the electrons as being degenerate (but not relativistic), then

$$P_e = K\left(\frac{Z}{A}\right)^{5/3}\left(\frac{\rho}{1\ {\rm g\ cm^{-3}}}\right)^{5/3}$$
$$= 10^{13}\ {\rm erg\ cm^{-3}}(1/2)^{5/3}(2\times 10^{6})^{5/3} = 10^{23}\ {\rm erg\ cm^{-3}}$$

- Base of the envelope is degenerate at the start of the runaway.

XRB Burning

- CNO cycle once T > 10^7 K
- Hot CNO at higher T
  - p-captures are shorter than β+ decay
  - Energy generation rate becomes independent of T

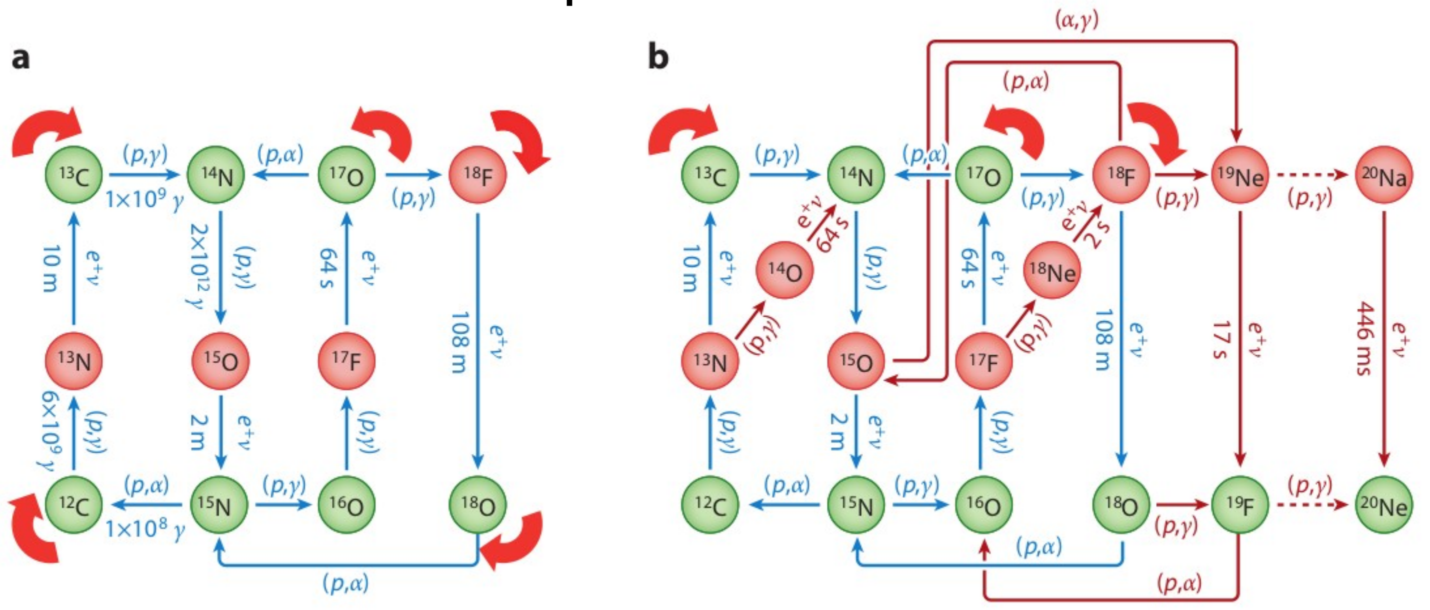


Figure 1

(*a*) The reaction flow in the three cold CNO cycles. (*b*) The hot CNO cycles, which represent a much more complex cyclic reaction pattern. Dashed lines indicate breakout reactions into the NeNa mass range, which occur at temperatures in excess of 0.4 GK.

(Wiescher et al. 2010)

XRB Bursts

- Pure He bursts are different
  - Energy release is rapid (no waiting on weak interactions)
  - Eddington limit is likely exceeded
  - Photosphere radius expansion burst can occur
    - These have become popular lately as a means to determine NS mass and radii
- H burning
  - Once we are hot enough, breakout reactions move us beyond hot CNO cycle
  - Proton captures build heavy nuclei
  - H can be exhausted before He burning is done: C can build up

XRB Burning

- As the burning proceeds, we can break out of the CNO cycle and build up proton-rich nuclei

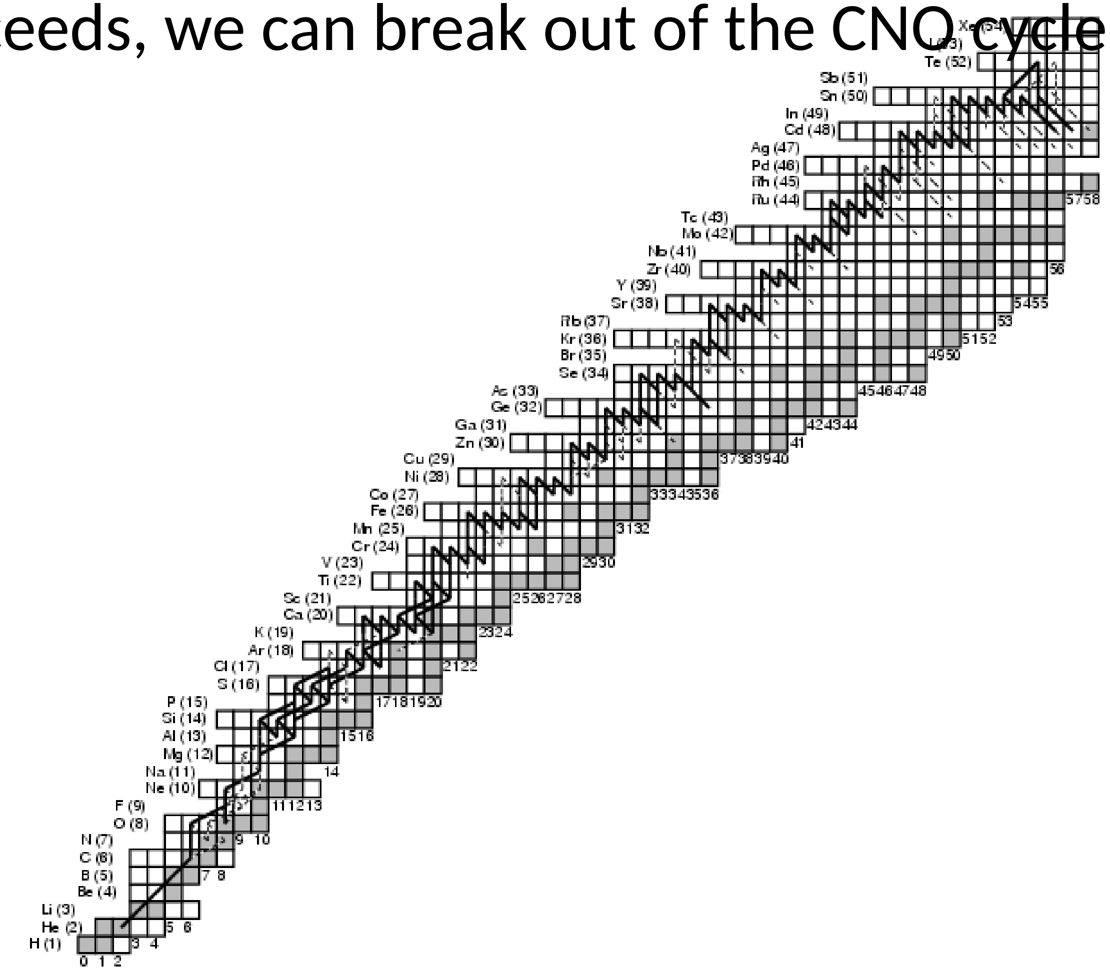
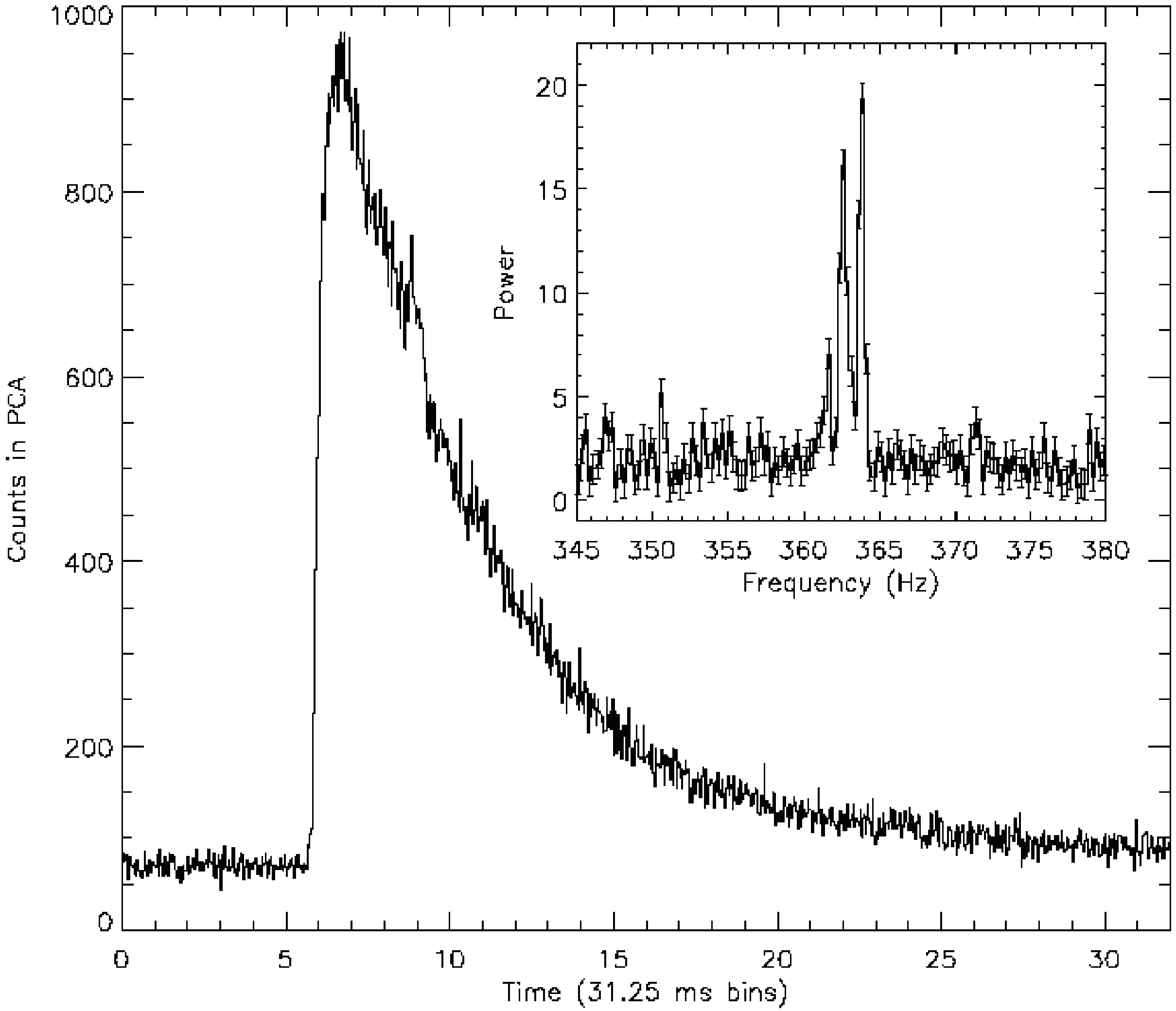


Fig. 3.1. Schematic showing the dominant pathways of the nuclear reaction flows during the rp process. Elements far beyond ^{56}Fe can easily be reached. Filled squares denote stable nuclides (after Schatz et al. 2001).

XRBs

- A pure He lightcurve



XRBs

- Multiple bursts from the same system

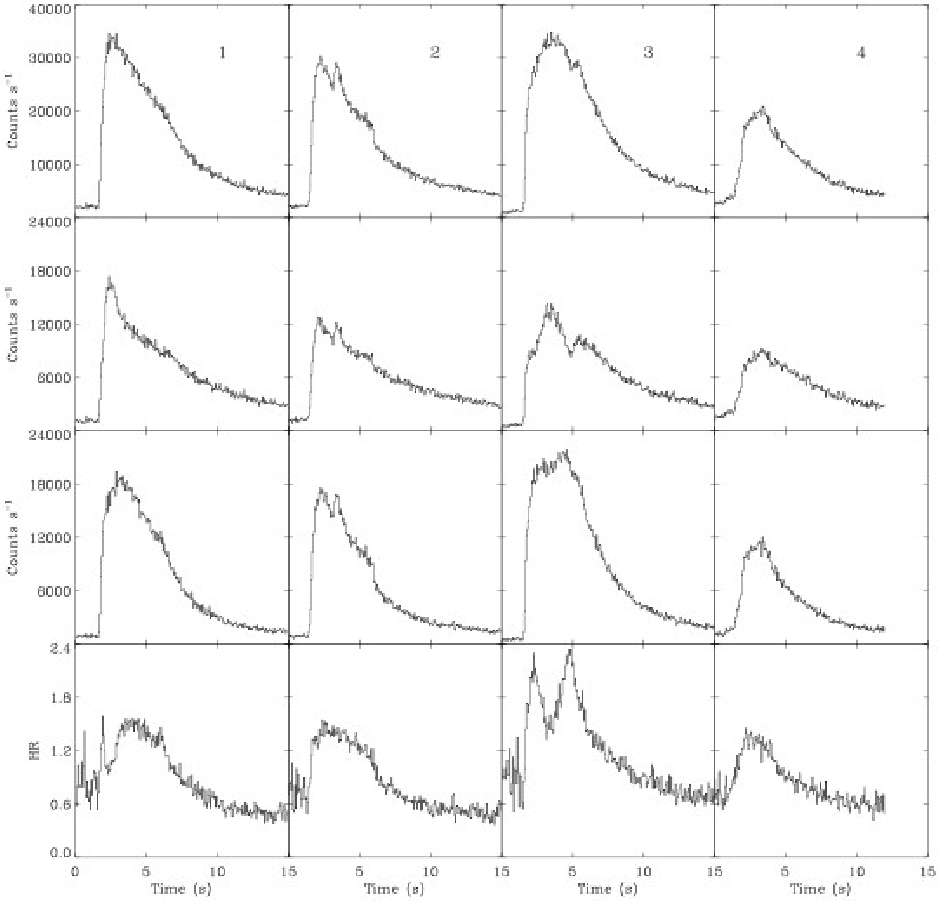
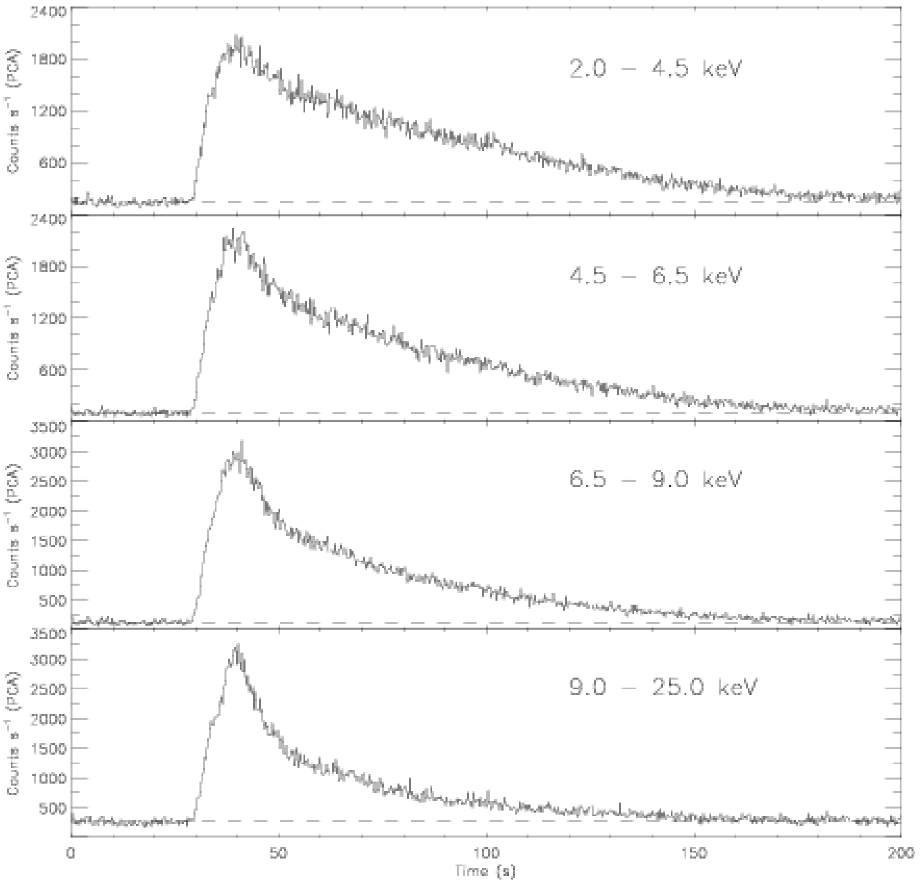


Fig. 3.4. A sample of four X-ray bursts from the LMXB 4U 1728–34 as observed with the RXTE/PCA. Each sequence shows, from top to bottom, the total 2 - 60 keV countrate, the 2 - 6 kev countrate, the 6 - 30 keV countrate, and the hardness ratio (6 - 30 keV) / (2 - 6 keV). Bursts 1 and 3 show clear evidence for PRE based on the hardness ratio evolution.

(Adapted from Strohmayer and Bildsten 2003)

XRBs

- A hydrogen burst—note the longer timescales because of the waiting points with H burning

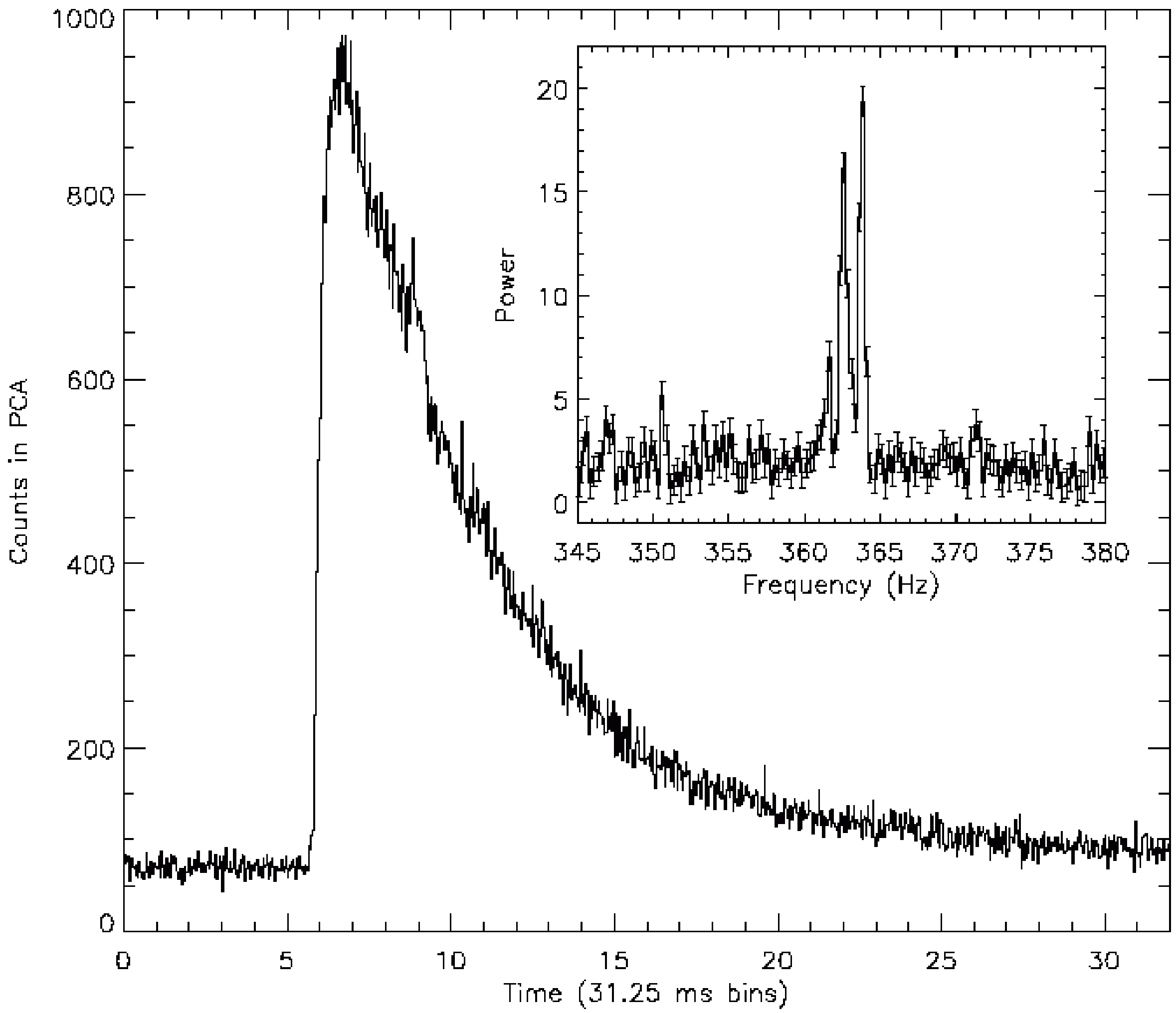


(from Strohmayer and Bildsten 2003)

Fig. 3.3. An X-ray burst from GS 1826–238 seen with the RXTE/PCA. The burst is shown in four different energy bands. The long duration is indicative of the delayed energy release from the rapid proton (rp) process. The dashed line marks the preburst flux level (see also Kong et al. 2000).

XRB Observations

- Light curve has a fast rise
- Decay is slower—this is the thermal diffusion timescale
- X-ray luminosity can be at the Eddington limit
  - Photosphere can lift off
- Brightness oscillations are observed (300 to 600 Hz)
  - Evident in power spectrum of lightcurve
  - Spin must be at play here
  - Evidence for non-uniform burning—perhaps localized ignition?



Strohmayer et al., 1996, ApJ, 469:L9

XRB Observations

- Oscillations can be observed in the rise of the burst
  - Amplitude is higher when X-ray flux is lowest
  - Likely due to small hot spot spreading across the entire NS.
- Oscillations during decay
  - Usually smaller amplitude than during rise
  - Some bursts show oscillations both during rise and decay
  - Not clear how to explain with the spreading hot spot idea
- Frequency changes during bursts
  - Frequency (usually) increases during the burst to some limiting value
  - Cause: angular momentum conservation expanded shell contracting back to the NS surface (?)
    - Can't account for all of the observed frequency increase

XRB Frequency Evolution

3.4 Millisecond variability during X-ray bursts **23**

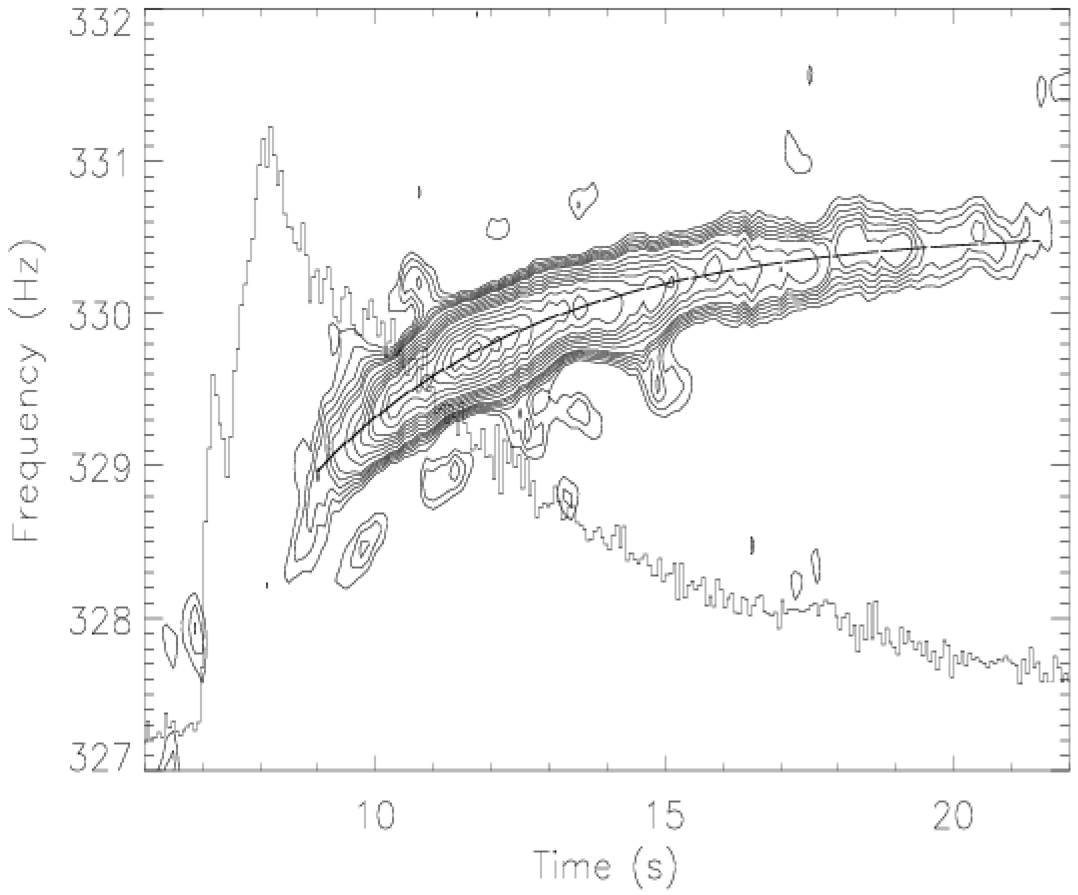


Fig. 3.9. An X-ray burst from 4U 1702–429 observed with the PCA onboard RXTE. Shown are contours of constant power spectral density as a function of frequency and time. The solid curve shows the best fitting exponential model. The burst time profile is also shown (after Strohmayer & Markwardt 1999).

(from Strohmayer and Bildsten 2003)

Superbursts

- Superbursts: X-ray flash lasts for hours instead of 10s of seconds
  - Thermonuclear in nature
  - Last 1000x longer and produce more energy than XRBs
- Best model—nuclear burning at higher densities (more fuel)
  - Assume nuclear energy release of 0.3 MeV per baryon
  - Observed energy of 10^{42} erg means 3.5×10^{24} g = 2 years of accretion (at 10^{-9} solar masses / year)
- Very long duration (several hours)
- Most show precursors
  - Burst in deep layers ignites H/He causing standard burst
- Best model—carbon ignition below the accreted H/He layer—this is the leftover C ash

Superbursts

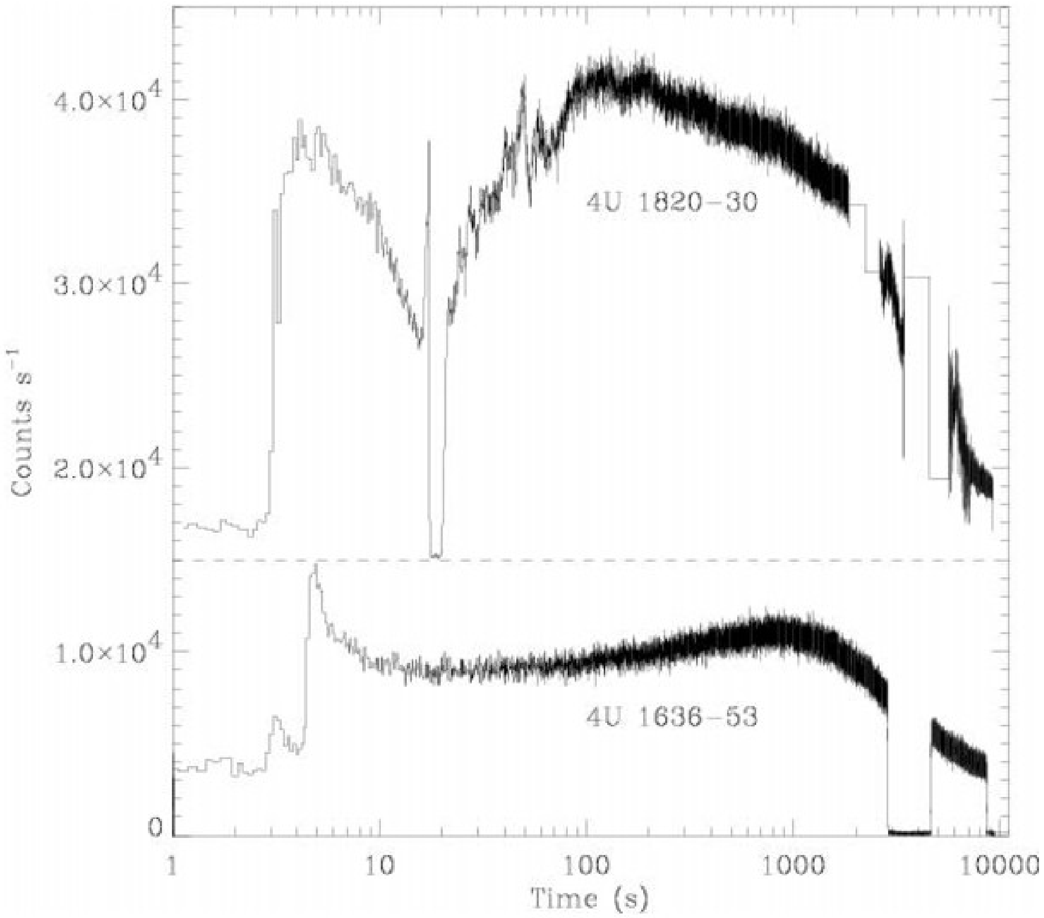


Fig. 3.15. Two superbursts observed with the RXTE/PCA. Shown are the 2 - 30 keV count rate histories observed in the PCA. Note the shorter precursor events prior to the superbursts. The event from 4U 1820–30 has been displaced vertically for clarity. The horizontal dashed line shows the zero level for this event. The time axis is logarithmic (after Strohmayer & Brown 2002; Strohmayer & Markwardt 2002).

(from Strohmayer and Bildsten 2003)

Modeling XRBs

- Most of what we know comes from 1-d (or even 1-zone) models
  - Able to use large networks to explore the nucleosynthesis
- Multi-d simulations show that rotation is important

Summary of XRB Calculations

- 1-d models successfully reproduce multiple bursts, get the recurrence time right, etc. (see, e.g. Woosley et al. 2004)
  - Spherically symmetric
  - Simple approximation of convection
- 2-d shallow water hydro calculations show the importance of rotation in confining the burning
- Some progress modeling pure He bursts with low Mach algorithms (Lin et al. 2006 and Malone et al. 2011)
  - Differing approaches show differences in dynamics, resolution requirements, etc.
- Recent calculations using simplified hydrostatic vertical structure and high-aspect ratio zones showed effects of rotation on flame speed (Cavecchi 2012)
  - Doesn't capture turbulence interactions, not really hydrodynamics vertically

1-d XRB Models

- Woosley et al. (2004)
  - 1-d stellar evolution (Kepler code)
  - Used up to 1300 isotopes
  - Explored sensitivity of lightcurve to accretion rate, metalicity, and nuclear physics details
  - Found that not all H, He, and C is burned in outer layers and that some burning continues between bursts
    - Next burst lights in these ashes and is less energetic
  - Carbon left behind in pure He bursts
    - But amount seems too small for superbursts

1-d XRB Models

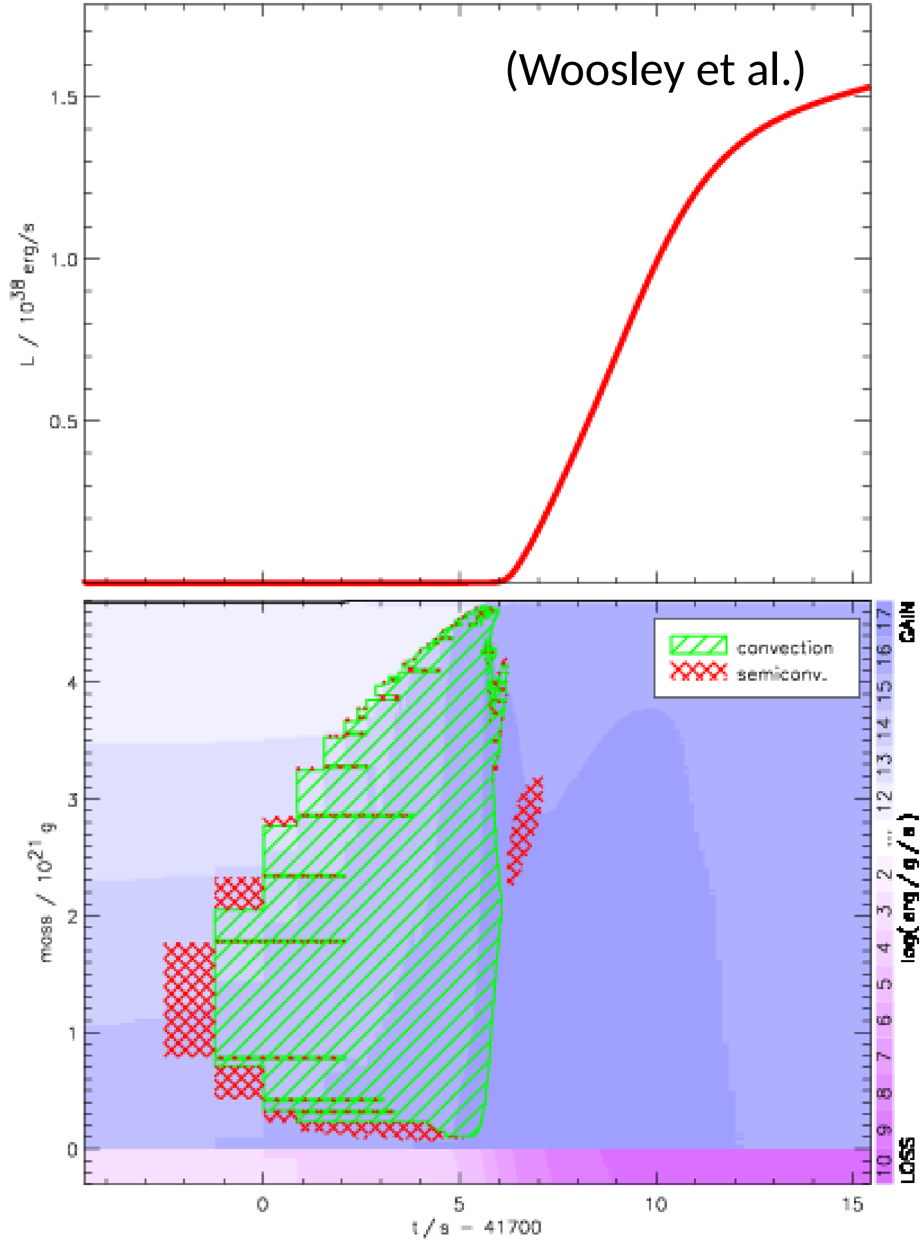


Fig. 4.— The first burst in Model zM. The lower panel shows that ignition initially occurs above the composition interface, but grows over the next 6s, both outwards and inwards, to encompass the entire shell. *Green hatched* regions are convective; *red cross hatched* regions are semi-convective. The observed burst commences shortly after convection reaches the surface and has begun to recede so that the luminosity during the rise is chiefly transported by diffusion. Times are offset by 41,700s since the beginning of accretion. Any effects due to the spreading of burning over the surface of the neutron star are ignored. Though given here for our standard choice, the rise time is sensitive to the nuclear reaction rates employed in the calculation (Fig. 20). In this and all subsequent depictions of the light curve, general relativistic effects have been ignored (§ 4.4).

Zingale

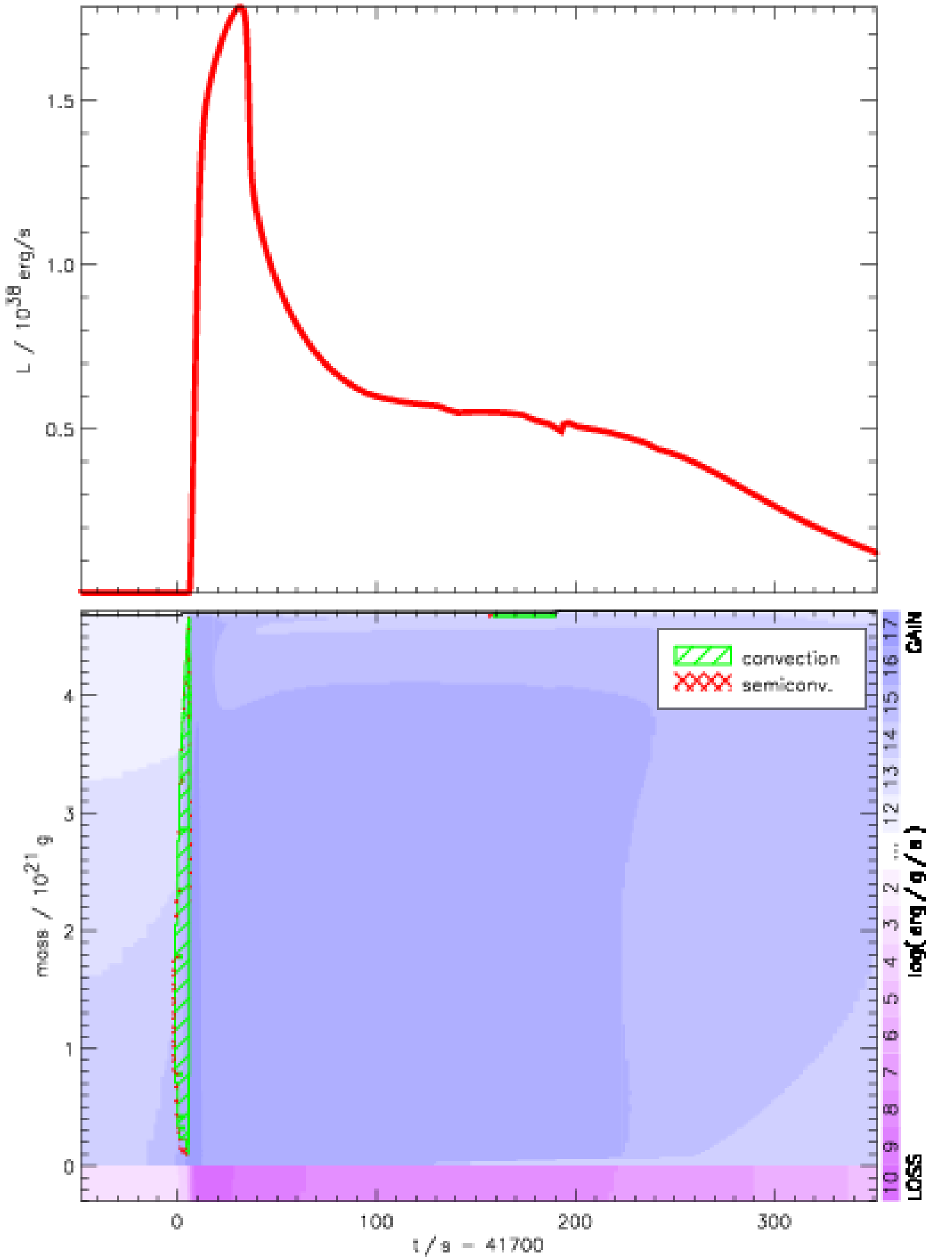


FIG. 5.— The top figure shows the light curve of burst 1 of Model zM. Following the main pulse, lasting about a minute, there is a long tail, lasting perhaps 5 min, powered by the continued burning of hydrogen. The shape of the light curve and its tail are sensitive to nuclear decay rates and proton capture along the rp-process path (Fig. 18). The bottom panel shows that appreciable convection only occurs prior to the rise of the pulse (see also Fig. 4). Shades of blue color indicate the nuclear energy generation rate while shades of purple indicate energy loss by neutrinos, both on a logarithmic scale. The flow of heat into the neutron star substrate is apparent.

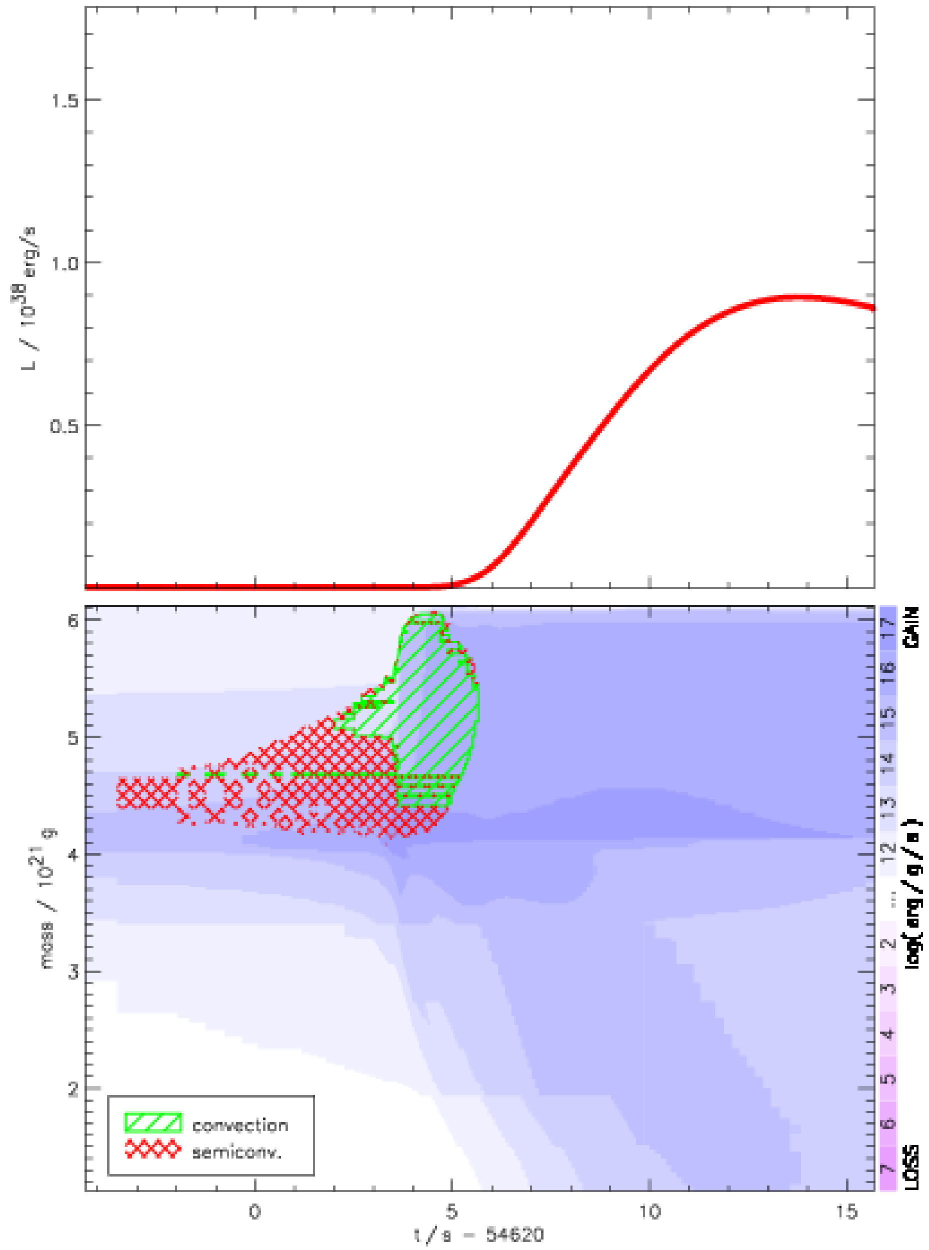


Fig. 10.— Rise time and convection for burst number 2 of Model zM. This is a more typical burst for the model than the one shown in Fig. 4. Once again, convection has ceased by the time the burst first becomes visible. Following a brief convective stage lasting about 2 s, well above the bottom of the freshly accreted layer, the principal burning starts inside the ashes of the previous burst which end at 4.7×10^{21} g (Fig. 9). The intensity of the blue scale indicates nuclear burning. Note that appreciable burning occurs in the ashes of burst number 1 as the heat wave from burst number 2 propagates through. This behavior is in contrast to the neutrino losses which dominated in Fig. 4. The rise time is sensitive to the nuclear reaction rates as well as the diffusion time scale (Fig. 20).

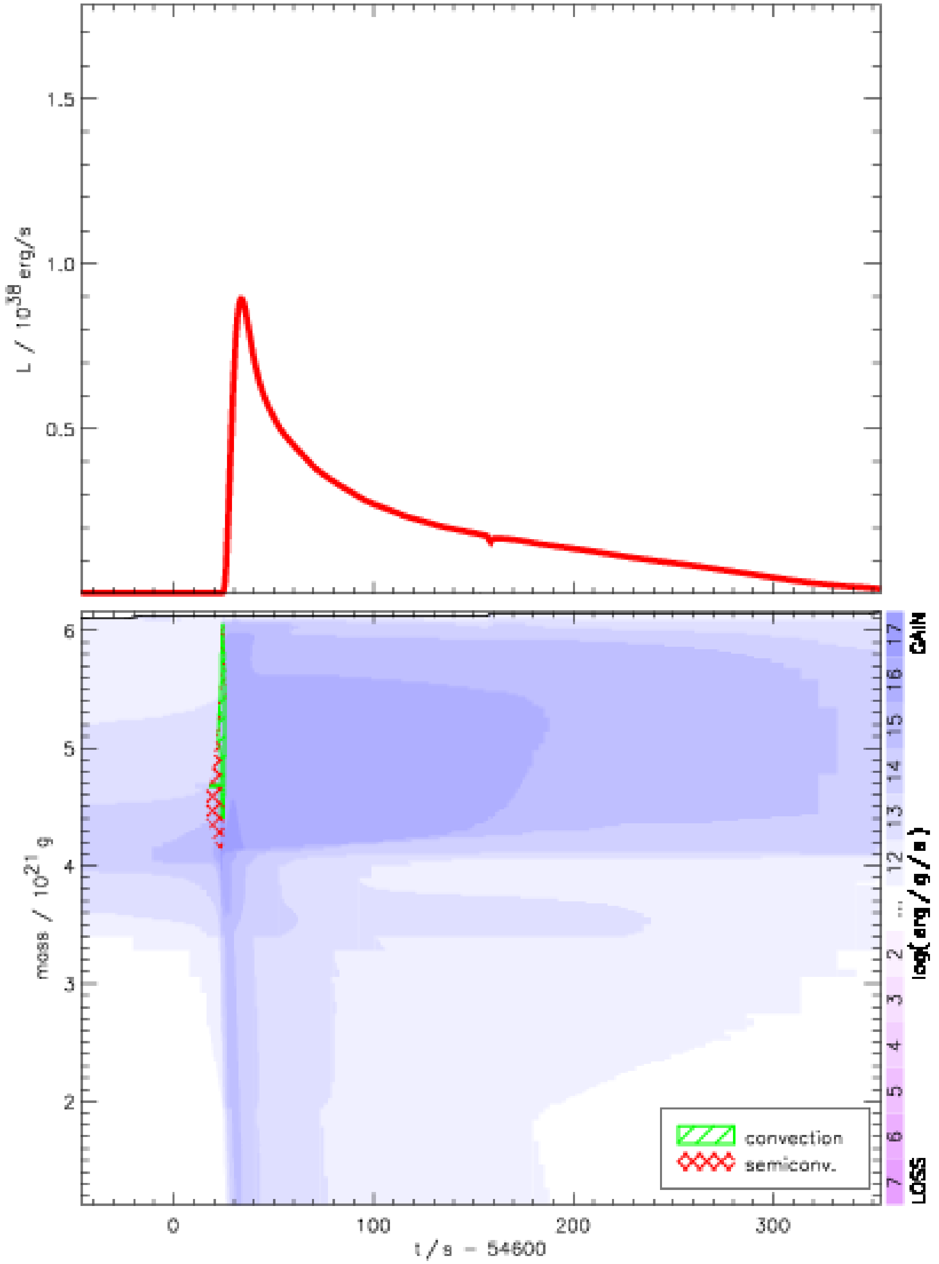


Fig. 11.— Light curve for burst number 2. This briefer, less luminous second burst is more typical of all subsequent bursts in Model zM. Shades of blue indicate nuclear burning.

(Woosley et al.)

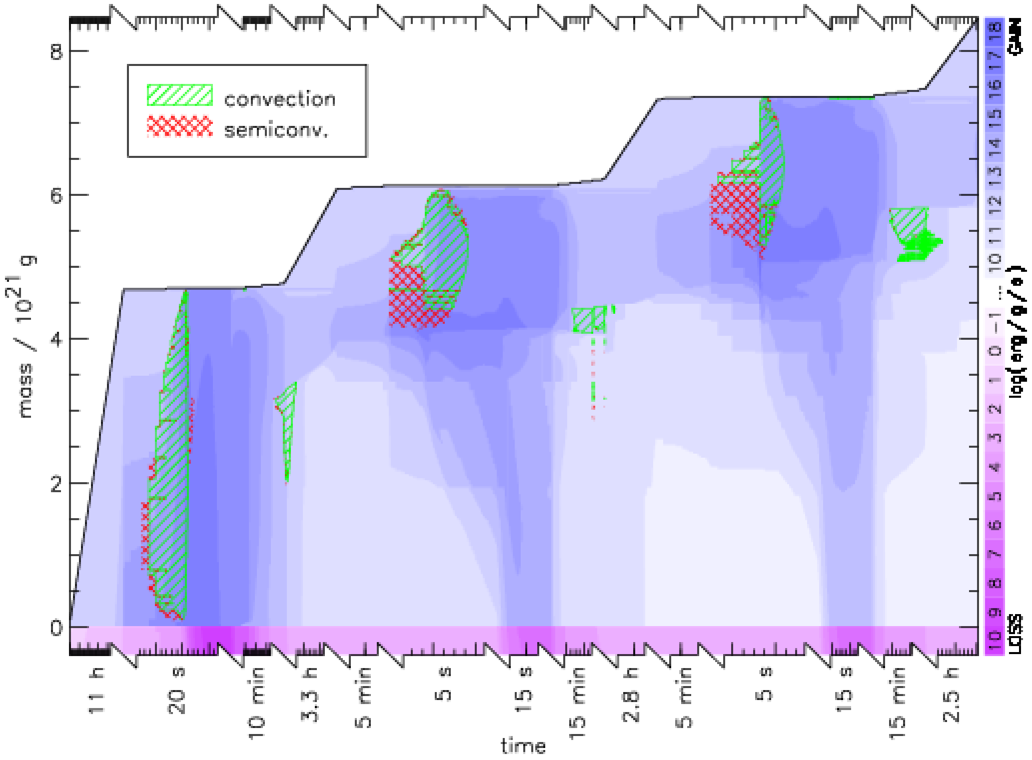


FIG. 14.— Kippenhahn diagram for bursts 1, 2, and 3 in Model zM. Green *hatching* indicates convection; *red cross hatching* semi-convective regions; *blue shading* shows net energy generation (nuclear minus neutrino losses); and pink shading indicates net energy losses (neutrino losses exceeding nuclear energy generation). Each level of blue color indicates an increase by one order of magnitude. The *y*-axis gives the enclosed mass coordinate above the assumed neutron star substrate and the thick black line gives the total mass of accreted material. The *x*-axis gives time increasing from left to right, but different parts of the evolution are plotted on different time scale. Breaks on the axis indicate a change of time scale and and below each segment we give the length of that time interval (not the total time). The rate of change of the total mass is inversely proportional to the magnification of time in each section and is thus also and indicator of the evolutionary time scale in each section. Note repeated waves of nuclear burning in the ashes of previous bursts as heat from the current burst propagates inwards. There are also periods of convection in between bursts.

Zingale

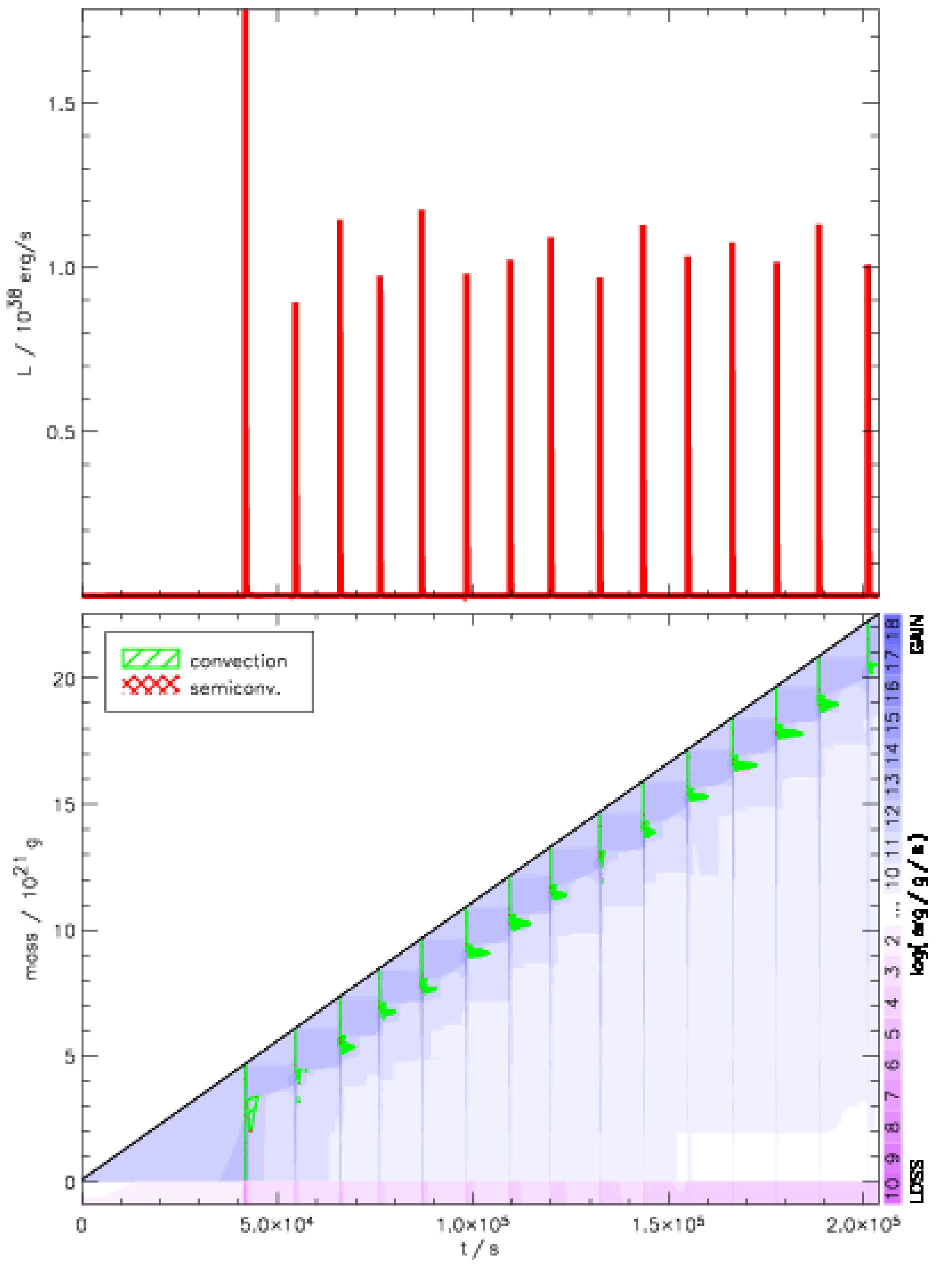
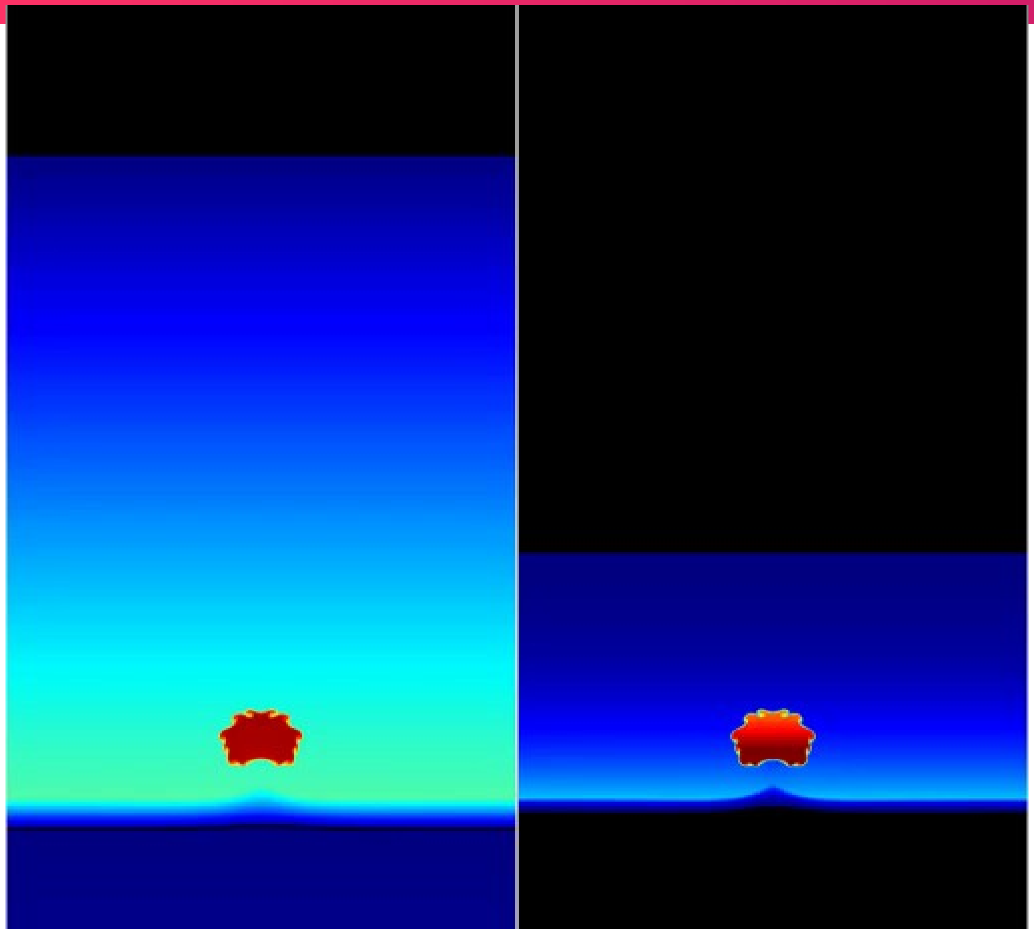


Fig. 15.— Fourteen flashes from Model zM. Note the regularity of the last 13. The first is a start up transient. Note also the heat flow and burning in the ashes of earlier bursts.

Multi-d XRB Models

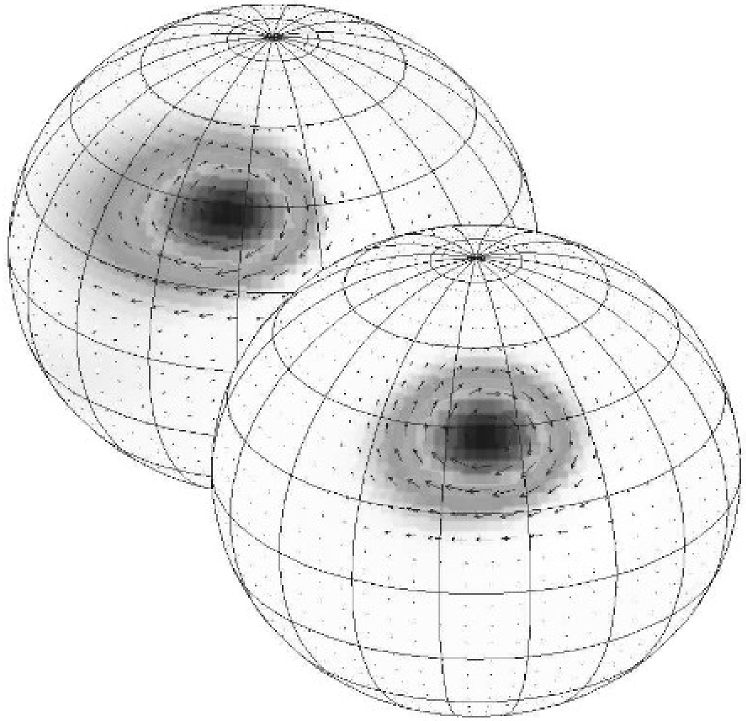


MAESTRO simulation (L = 3.84 m, Δx = 0.5 cm) evolved to t = 1.5×10^{-5} s showing dissiapation of a hot spot (initial T = 10^9 K) in a He NS accreted layer. Temperature is shown on the left, nuclear energy generation rate is shown on the right.

- Atmosphere is only partially degenerate, so it is hard to localize a hot spot
  - Hot spot fizzles out
  - May cause some stirring
  - Similar to novae (Shankar & Arnett 1994)

Multi-d XRB Models

- Rotation is crucial
  - Geostrophic balance: Coriolis force balances the lateral spreading
- Spitkovsky et al. (2002)
  - Shallow water wave calculations
  - Coriolis force can balance the outward spreading
  - Hotspot spreads across the surface of the NS

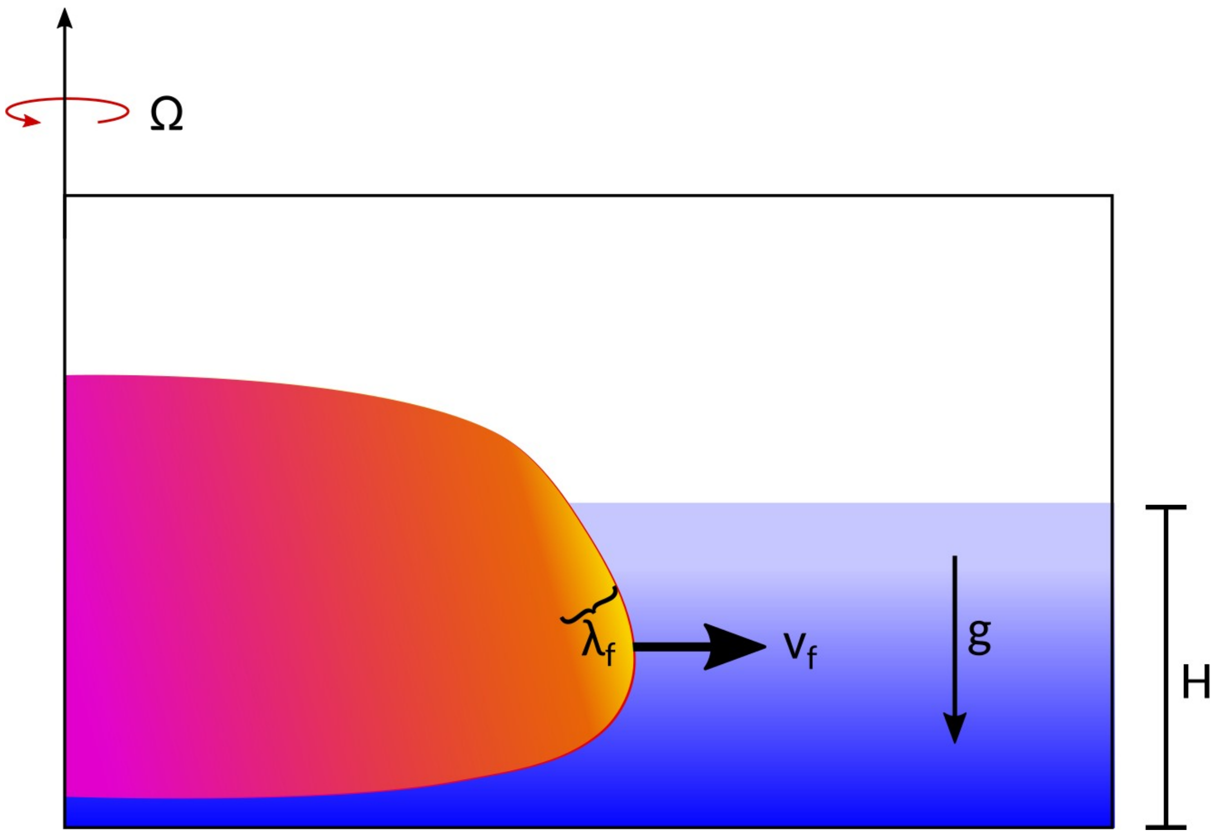


(Adapted from Strohmayer and Bildsten 2003)

Fig. 3.13. Initial evolution of a burning hot spot ignited off the equator as seen in a frame rotating with the neutron star. Velocity vectors show the circulation of the fluid induced by the Coriolis forces. The hot spot expands due to burning and drifts west-southwest because of the latitude dependence of the Coriolis force (after Spitkovsky, Levin & Ushomirsky 2002).

Multi-d XRB Models

- Geostrophic balance: lateral spread balanced by Coriolis force
- Flame width ~ pressure scale height



What Can We Learn In Multi-D

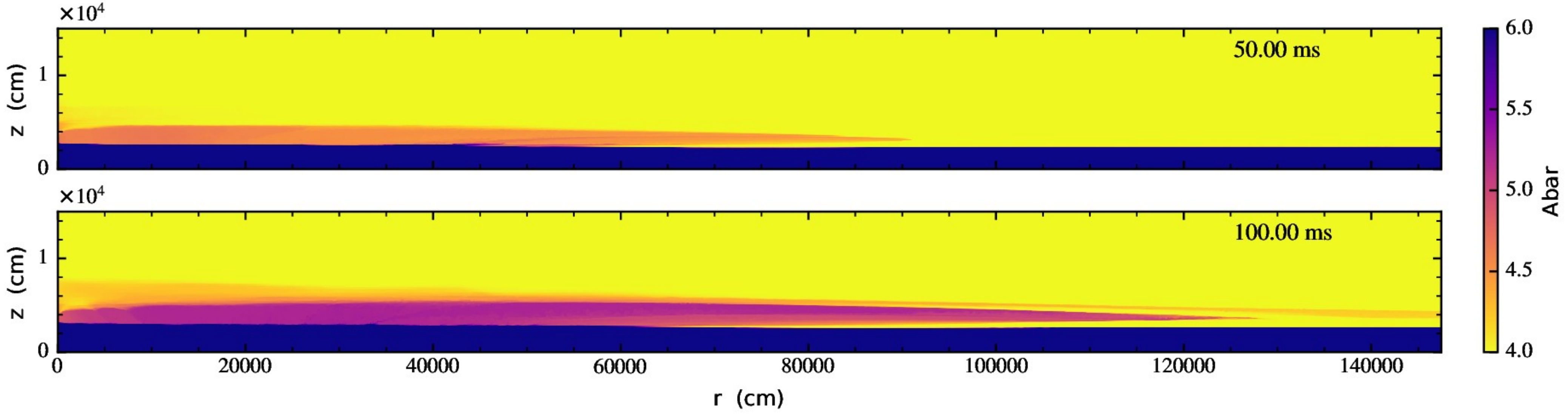
- How does the fuel spread over the surface?
- How does the ignition begin?
  - Convection is likely important in the moments leading up to the ignition — this is a 3-d problem.
  - How many locations does the burning begin at?
- Is the burning localized?
- If so what localizes it?
- How does it spread?
- How fast does the burning spread?
- Does convection modify the nucleosynthesis?
- What are the effects of rotation?

XRB Challenges

- Length scales
  - Flame is ~20 cm thick
  - Pressure scale height ~100 cm
  - Balance of flame spreading and Coriolis force ~ 10^5 cm
  - NS circumference ~ 6×10^6 cm
- Time scales
  - Accretion ~ hours to days
  - Convection ~ minutes
  - Rise time ~ 1 s
  - Diffusion timescale ~ 10s
- Computational timestep:

$$\Delta t \approx \frac{\Delta x}{c_s} \approx \frac{10 \text{ cm}}{5 \times 10^8 \text{ cm/s}}$$
$$\approx 2 \times 10^{-8} \text{ s}$$

Modeling Flame Spreading



(Harpole et al. 2021)

Modeling Flame Spreading

- Calculations show:
  - Flame accelerates ~ 10× over laminar speed
  - Ash on surface leads front
  - Hotter models show acceleration
- Velocity ~10^6 cm/s is almost consistent with rise times
- Currently running: 3-d model, mixed H/He bursts