PHY 521: Stars

Homework #3

Due: 2025-10-02 (by the end of the scheduled class period)

Be sure to show all work. Assignment is turned in via Brightspace assignment upload. You should make a single, legible PDF with your solutions. You must show your work and include units with all numbers, all throughout each problem.

While you are free to discuss problems with your classmates, your work must be your own. Any use of AI/LLM must be disclosed. All calculations (including any code you may write) and writing up of the problem solution must be your own work.

Unless otherwise noted, all problems carry equal weight.

1. *LTE?* In the interior of a star, we are assuming that we are in LTE. But what about the atmosphere? Imagine the very outer layers of the Sun—you can take ρ and g (the gravitational acceleration near the surface) as constant.

   (**a**) Assuming a solar composition and that the ideal gas applies, solve the equation of hydrostatic equilibrium near the surface and show that the density has a form:

$$\rho(r) = \rho_0 e^{-r/H} \tag{1}$$

   where ρ_0 is the density at some reference height $r = 0$. Here H is the scale height. Evaluate it for the photosphere (take $T = 6000$ K).

   (**b**) Estimate the mean free path if we take a photosphere density of $\rho = 2.5 \times 10^{-7}\ \mathrm{g\ cm^{-3}}$ and an opacity of $\kappa = 0.26\ \mathrm{cm^2/g}$.

   (**c**) comparing the mean free path and scale height, is LTE justified near the photosphere?

2. *Limb darkening.* (based on Choudhuri) Looking at the disk of the Sun, you will notice that the center appears bright while the perimeter is darker—a phenomenon called *limb darkening*. Here we will derive a model for the intensity as a function of angle for the solar disk.

Recall that in radiative equilibrium, the average intensity $\langle I \rangle \sim S$. We found an expression in class for the radiation pressure:

$$P_\gamma(\tau) = \frac{F}{c}\left(\tau + \frac{2}{3}\right) \tag{2}$$

where F is the radiative flux. Using the expression $P = U/3$, and the relation between the energy density U and $\langle I \rangle$, express the source function, S, as a function of τ.

Now, consider the general solution to the transfer equation for the case where $\mu \geq 0$ (outgoing radiation). Find the intensity as a function of μ at $\tau = 0$ for this source function and sketch $I(\tau = 0, \mu)/I(\tau = 0, \mu = 1)$—this is the change in intensity as we move from the center of the solar disk to the limb.

3. *Atmospheres.* (from C&O) Consider a horizontal plane-parallel slab of gas of thickness L that is maintained at a constant temperature T. Assume that the gas has an optical depth $\tau_{\lambda,0}$ with $\tau_\lambda = 0$ at the top surface of the slab. Assume further that incident radiation of intensity $I_{\lambda,0}$ enters the bottom of the slab from outside.

Use the general solution of the transfer equation we derived in class,

$$I(\tau, \mu) = e^{-(\tau_0 - \tau)/\mu} I(\tau_0, \mu) - \int_{\tau_0}^{\tau} e^{-(t-\tau)/\mu} \frac{S}{\mu} dt \tag{3}$$

to show that when you are looking at the slab from above ($\mu = 1$):

(**a**) you see blackbody radiation if $\tau_{\lambda,0} \gg 1$

(**b**) if $\tau_{\lambda,0} \ll 1$, you see absorption lines superimposed on the spectrum of the incident radiation if $I_{\lambda,0} > S_\lambda$, and emission lines superimposed on the spectrum of the incident radiation if $I_{\lambda,0} < S_\lambda$.

(*Hint: you may assume that the source function, S_λ does not vary with position in the slab. You may also assume LTE when $\tau_{\lambda,0} \gg 1$.*)